which they might be distinguished from the known particles? Since the mass difference Δ cannot be estimated at present, we can only speculate about possible decay modes of p_2. It might, e.g., decay by a β interaction: $p_2 \rightarrow n_1 + e_{1,2}^{+} + \nu$. Other weak decay modes might exist; in particular, one or more π mesons or K mesons might be emitted if Δ were sufficiently high. Since p_2 must be assumed to have also strong interactions, though not in decay, it may occur bound to other, ordinary, nucleons, and its decay in such a state may resemble a hyperfragment disintegration.[7] The same holds for $\bar{p}_2$, which could either annihilate "slowly" inside a nucleus through its weak interaction, e.g., $\bar{p}_2 + n_1 \rightarrow e_{1,2}^{-} + \bar{\nu}$, or first decay into $\bar{p}_1$ or $\bar{n}_1$ with consequent rapid annihilation.

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[7]From time to time there have been reports of cosmic-ray events observed in photographic emulsions which appear to show a Q value larger than expected from hyperon decay. In the light of the question raised above, some of these events might be worth re-analyzing. See, e.g., Y. Eisenberg, Phys. Rev. 96, 541 (1954); Fry, Schneps, and Swami, Phys. Rev. 97, 1189 (1955), and Nuovo cimento 2, 346 (1955); Castagnoli, Cortini, and Franzinetti, Nuovo cimento 2, 550 (1955).

RADIATIVE CORRECTIONS TO PION BETA DECAY*

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Recent experiments[1] have shown that the ratio R_0 of the rates Γ_e, Γ_μ for $\pi \rightarrow e + \nu$ and $\pi \rightarrow \mu + \nu$ modes of decay is in the neighborhood of the value predicted by the universal beta-decay theory,[2,3] i.e., $R_0 = (12.78 \times 10^{-5})$. However, to compare the results of a precise experiment with the predicted number the radiative corrections should be included in the theoretical estimate of the ratio. The corrections do not cancel in the ratio since they depend on the electron and muon masses and, furthermore, produce a surprisingly large correction to the ratio.

Using the universal (V, A) theory[3] we express the general matrix element for the process $\pi \rightarrow (e \text{ or } \mu) + \nu + \gamma$ to order e as

$$M_\nu = [f_1 \delta_{\mu\nu} + f_2 k_\mu p_\nu + f_3 p_\mu p_\nu] \times \bar{\Psi}_{l_2} \gamma_\mu (1 + i\gamma_5) \Psi_{l_1} \cdots \quad , \qquad (1)$$

where p and k are the momentum of the pion and photon, respectively, and where each of the covariants f_1, f_2, f_3 depend on the scalars m_π^2, $p \cdot k$, k^2. In addition to M_ν we also consider the matrix elements which arise from bremsstrahlung photons. In writing Eq. (1) we have used the Lorentz condition to eliminate terms such as $k_\nu p_\mu$, $k_\nu k_\mu$. We note that gauge invariance implies that f_1 at $k=0$ is determined by the amplitude for $\pi \rightarrow (e \text{ or } \mu) + \nu$.

In the phenomenological theory with direct π-(e or μ) coupling only f_1 would be present in Eq. (1). In a realistic theory involving virtual nucleon loops, the other covariants f_2, f_3 would be expected to enter. If we consider the decay of the pion to occur through one virtual nucleon pair then we find that $f_{2,3} \sim (1/M^2) f_1$ where M is the nucleon mass and that in the limit of very large nucleon mass f_1 does not depend on $p \cdot k$ and k^2. If we consider $(k^2/M^2) \ll 1$ then application of the Ward identity leads to the same conclusions to all orders in the pion-nucleon coupling. We therefore have neglected $f_{2,3}$ in the calculation of the radiative corrections, and have assumed f_1 is independent of k. In this case f_1 will cancel out in the ratio of the rates.

Using the first term of Eq. (1) we have calculated the radiative corrections, in the usual manner, by including all possible real and virtual electromagnetic processes to order e^2. Since the inclusion of the inner bremsstrahlung changes the number of particles in the final states from two to three there is now a spectrum of energies available to the electron or muon. If we ask how the ratio of rates is affected for electrons or muons having an energy near the maximum energy and suffering an energy loss less than

ΔE, then we obtain for the new ratio R, in units of $\hbar = c = 1$ and $e^2 \approx 1/137$,

$$R=(12.78\times10^{-5})\Big[1-\frac{2e^2}{\pi}\Big\{\tfrac{3}{4}\ln(m_\mu/m_e)+\ln(2\Delta E_e/m_e)-\ln(2\Delta E_\mu/m_e)+\left(\frac{m_\pi^2+m_e^2}{m_\pi^2-m_e^2}\right)\times\ln(m_\pi/m_e)\ln\left(\frac{m_\pi^2-m_e^2}{2\Delta E_e m_\pi}\right)-\left(\frac{m_\pi^2+m_\mu^2}{m_\pi^2-m_\mu^2}\right)\times\ln(m_\pi/m_\mu)\ln\left(\frac{m_\pi^2-m_\mu^2}{2\Delta E_\mu m_\pi}\right)-\frac{m_e^2}{m_\pi^2-m_e^2}\times\ln(m_\pi/m_e)+\frac{m_\mu^2}{m_\pi^2-m_\mu^2}\ln(m_\pi/m_\mu)-\ln\left(\frac{m_\pi^2-m_e^2}{m_\pi^2-m_\mu^2}\right)\Big\}\Big], \quad (2)$$

or

$$R=[11.00+0.27\ln(2\Delta E_e/m_e)]\times10^{-5}, \quad (3)$$

where ΔE_e and ΔE_μ are the energy intervals below the maximum energy over which electron or muon counts are being accepted[4]. We have omitted from Eq. (3) the term which depends on $\ln(\Delta E_\mu/m_e)$ since it is completely negligible. For $2\Delta E_e = m_e \approx 0.5$ Mev the radiative corrections amount to a surprisingly large decrease in the ratio of amount 14%.

We note that even though the rate for $\pi \to (\mu$ or $e) + \nu$ contains logarithms of the ultraviolet cutoff, the ratio of rates will not depend on the cutoff as long as we take it to be the same for each of the decay modes.[5] Furthermore, the contributions to Eq. (3) from virtual processes come almost entirely from very low photon momenta. If this were not true, but instead the contributions to R came from regions of large virtual photon momenta, then the radiative corrections might depend on the terms in f_1 dependent on $p\cdot k$ and k^2 and also on the other covariants.

The results given here do not agree with a recent "theorem" by Gatto and Ruderman.[6] However using a theorem due to Ruderman and Watson[7] a closely related statement can be made: Let Γ_P and Γ_A represent the rates for one of the pion decay channels with the leptons coupled with either P or A interaction. Then the ratio Γ_P/Γ_A including electromagnetic effects, expressed in terms of the bare masses, is equal to the ratio Γ_P/Γ_A without including electromagnetic effects. In particular for the electron decay mode $\Gamma_P/\Gamma_A = (m_\pi^{(b)}/m_e^{(b)})^{-1}$ with and without including radiative corrections. If we are interested in

$$\left(\frac{\Gamma_e}{\Gamma_\mu}\right)_A=\left[\frac{(\Gamma_e/\Gamma_\mu)_A}{(\Gamma_e/\Gamma_\mu)_P}\right]\left(\frac{\Gamma_e}{\Gamma_\mu}\right)_P ,$$

then we note that the term in brackets is equal to the ratio of the square of the bare masses of the electron and muon with or without including conventional electrodynamics. However, one should also note that $(\Gamma_e/\Gamma_\mu)_P$ receives a large radiative correction which to first order in e^2 is around 11% decrease.

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[4]Since the neutrino is not observed we have included in Eq. (2), in the calculations of the inner bremsstrahlung, the contributions from all photons having momenta consistent with the conservation laws. This means that in the integration over real photon momenta the maximum photon energy will be a function of x, the cosine of the angle between electron (muon), and of the energy interval ΔE, i.e.,

$$\omega_{\max}=\left(\frac{2m_\pi^2}{m_\pi^2-m_e^2}\right)\left(\frac{\Delta E}{1+x}\right).$$

[5]If, for example, we take the cutoffs occurring in the virtual processes to be proportional to the respective masses of electron or muon, then R increases by about 1.5% over that given by Eq. (3).

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MUON K-CAPTURE COMPARED TO β DECAY FOR $C^{12} \to B^{12}$

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Recent refinements of the theory of the V-A universal Fermi interaction[1] have made it desirable to attempt a more precise experimental