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The Standard model prediction for $K_{e2}/K_{\mu 2}$ and $\pi_{e2}/\pi_{\mu 2}$

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We have calculated the ratios $R^{(P)}_{e/\mu} \equiv \Gamma(P \to e\bar{\nu}_e[\gamma])/\Gamma(P \to \mu\bar{\nu}_\mu[\gamma])$ ($P = \pi, K$) in Chiral Perturbation Theory up to $\mathcal{O}(e^2 p^4)$, finding $R^{(\pi)}_{e/\mu} = (1.2352 \pm 0.0001) \times 10^{-4}$ and $R^{(K)}_{e/\mu} = (2.477 \pm 0.001) \times 10^{-5}$. This observable is helicity suppressed in the Standard Model, so that it is a sensitive probe of all Standard Model extensions that induce pseudoscalar currents and nonuniversal corrections to the lepton couplings. Ongoing experimental searches plan to reach uncertainties that are comparable to these results. At the moment $R^{(K)}_{e/\mu}$ is in agreement with the final result by the KLOE Collaboration at DAFNE and it is at $1.4\,\sigma$ of the preliminary result by the NA62 Experiment at CERN. New measurements of $R^{(\pi)}_{e/\mu}$ are under way by the PEN Collaboration at PSI and by the PIENU Collaboration at TRIUMF.

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1. Motivation

The ratio $R_{e/\mu}^{(P)} \equiv \Gamma(P \to e\bar{\nu}_e[\gamma])/\Gamma(P \to \mu\bar{\nu}_\mu[\gamma])$ $(P = \pi, K)$ of leptonic decay rates of light pseudoscalar mesons is helicity-suppressed in the Standard Model, due to the $V-A$ charged current coupling. It is therefore a sensitive probe of all Standard Model extensions that induce pseudoscalar currents and non-universal corrections to the lepton couplings [1]. Attention to these process has been payed in the context of the Minimal Supersymmetric Standard Model, with [2] and without [3] lepton-flavor-violating effects. In general, effects from weak-scale new physics are expected in the range $(\Delta R_{e/\mu})/R_{e/\mu} \sim 10^{-4} - 10^{-2}$ and there is a realistic chance to detect or constrain them because of the following circumstances:

i) Ongoing experimental searches plan to reach a fractional uncertainty of $(\Delta R_{e/\mu}^{(\pi)})/R_{e/\mu}^{(\pi)} \sim 5\times 10^{-4}$ [4, 5] and $(\Delta R_{e/\mu}^{(K)})/R_{e/\mu}^{(K)} \sim 3\times 10^{-3}$ [6, 7], which represent respectively a factor of around 5 and 10 improvement over former errors [8, 9, 10, 11, 12, 13].

ii) At the same time, the Standard Model theoretical uncertainty can be pushed below this level, since to a first approximation the strong interaction dynamics cancels out in the ratio $R_{e/\mu}^{(P)}$ and hadronic structure dependence appears only through electroweak corrections. Indeed, the most recent theoretical predictions read $R_{e/\mu}^{(\pi)} = (1.2352 \pm 0.0005)\times 10^{-4}$ [14], $R_{e/\mu}^{(\pi)} = (1.2354 \pm 0.0002)\times 10^{-4}$ [15], and $R_{e/\mu}^{(K)} = (2.472 \pm 0.001)\times 10^{-5}$ [15]. In Ref. [14] a general parameterization of the hadronic effects is given, with an estimate of the leading model-independent contributions based on current algebra [16]. The dominant hadronic uncertainty is roughly estimated via dimensional analysis. In Ref. [15], on the other hand, the hadronic component is calculated by modeling the low- and intermediate-momentum region of the loops involving virtual photons.

2. The Standard Model prediction

In Refs. [17, 18] we have analyzed $R_{e/\mu}^{(P)}$ within Chiral Perturbation Theory [19], the low-energy effective field theory of QCD. The key feature of this framework is that it provides a controlled expansion of the amplitudes in terms of the masses of pseudoscalar mesons and charged leptons ($p \sim m_{\pi,K,\ell}/\Lambda_\chi$, with $\Lambda_\chi \sim 4\pi F_\pi \sim 1.2\,\mathrm{GeV}$), and the electromagnetic coupling (e). Electromagnetic corrections to (semi)-leptonic decays of K and π have been worked out to $\mathcal{O}(e^2p^2)$ [20, 21], but had never been pushed to $\mathcal{O}(e^2p^4)$, as required for $R_{e/\mu}^{(P)}$ in order to match the experimental accuracy.

Within the chiral power counting, $R_{e/\mu}$ is written as:

$$R_{e/\mu}^{(P)} = R_{e/\mu}^{(0),(P)}\,(1+\Delta_{LL})\left[1+\Delta_{e^2p^2}^{(P)}+\Delta_{e^2p^4}^{(P)}+\Delta_{e^2p^6}^{(P)}+\ldots\right], \tag{2.1}$$

being $R_{e/\mu}^{(0),(P)}$ the well known tree-level expression:

$$R_{e/\mu}^{(0),(P)} = \frac{m_e^2}{m_\mu^2}\left(\frac{m_P^2-m_e^2}{m_P^2-m_\mu^2}\right)^2 . \tag{2.2}$$

	$(P=\pi)$	$(P=K)$
$\tilde{c}_2^{(P)}$	0	$(7.84\pm 0.07_\gamma)\times 10^{-2}$
$c_2^{(P)}$	$5.2\pm 0.4_{L_9}\pm 0.01_\gamma$	$4.3\pm 0.4_{L_9}\pm 0.01_\gamma$
$c_3^{(P)}$	$-10.5\pm 2.3_{\rm m}\pm 0.53_{L_9}$	$-4.73\pm 2.3_{\rm m}\pm 0.28_{L_9}$
$c_4^{(P)}(m_\mu)$	$1.69\pm 0.07_{L_9}$	$0.22\pm 0.01_{L_9}$

Table 1: Numerical values for $c_{2,3,4}^{(P)}$ and $\tilde{c}_2^{(P)}$, for $P=\pi,K$. The uncertainties correspond to the input values $L_9^r(\mu=m_\rho)=(6.9\pm 0.7)\times 10^{-3}$, $\gamma=0.465\pm 0.005$ [24], and to the matching procedure (m), affecting only $c_3^{(P)}$.

At the level of uncertainty considered here, one needs to include higher order long distance corrections [14] and their effect amounts to the factor $1+\Delta_{LL}$ in (2.1),

$$1+\Delta_{LL}=\frac{\left(1-\frac{2}{3}\frac{\alpha}{\pi}\log\frac{m_\mu}{m_e}\right)^{9/2}}{1-\frac{3\alpha}{\pi}\log\frac{m_\mu}{m_e}}=1.00055\ . \tag{2.3}$$

The leading electromagnetic correction $\Delta_{e^2p^2}^{(P)}$ corresponds to the point-like approximation for pion and kaon, and its expression is also well known [14, 20, 22]:

$$\Delta_{e^2p^2}^{(P)}\;=\;\frac{\alpha}{\pi}\left[F(\frac{m_e^2}{m_P^2})-F(\frac{m_\mu^2}{m_P^2})\right], \tag{2.4}$$

$$F(z)\;=\;\frac{3}{2}\log z+\frac{13-19z}{8(1-z)}-\frac{8-5z}{4(1-z)^2}\,z\log z-\left(2+\frac{1+z}{1-z}\log z\right)\log(1-z)$$
$$-\;2\frac{1+z}{1-z}Li_2(1-z)\ . \tag{2.5}$$

The structure dependent effects are all contained in $\Delta_{e^2p^4}^{(P)}$ and higher order terms, which are the main subject of Refs. [17, 18]. Neglecting terms of order $(m_e/m_\rho)^2$, the most general parameterization of the next-to-leading chiral contribution can be written in the form

$$\Delta_{e^2p^4}^{(P)}=\frac{\alpha}{\pi}\frac{m_\mu^2}{m_\rho^2}\left(c_2^{(P)}\log\frac{m_\rho^2}{m_\mu^2}+c_3^{(P)}+c_4^{(P)}(m_\mu/m_P)\right)+\frac{\alpha}{\pi}\frac{m_P^2}{m_\rho^2}\tilde{c}_2^{(P)}\log\frac{m_\mu^2}{m_e^2}\ , \tag{2.6}$$

which highlights the dependence on lepton masses. The dimensionless constants $c_{2,3}^{(P)}$ do not depend on the lepton mass but depend logarithmically on hadronic masses, while $c_4^{(P)}(m_\mu/m_P)\to 0$ as $m_\mu\to 0$. (Note that our $c_{2,3}^{(\pi)}$ do not coincide with $C_{2,3}$ of Ref. [14], because their C_3 is not constrained to be m_ℓ-independent and contains in general logarithms of m_ℓ.)

Let us note that the results for $c_{2,3,4}^{(P)}$ and $\tilde{c}_2^{(P)}$ depend on the definition of the inclusive rate $\Gamma(P\to\ell\bar{\nu}_\ell[\gamma])$. The radiative amplitude is the sum of the inner bremsstrahlung (T_{IB}) component of $\mathcal{O}(ep)$ and a structure dependent (T_{SD}) component of $\mathcal{O}(ep^3)$ [23]. The experimental definition of $R_{e/\mu}^{(\pi)}$ is fully inclusive on the radiative mode, so that $\Delta_{e^2p^4}^{(\pi)}$ receives a contribution from the interference of T_{IB} and T_{SD}. Moreover, in this case one also has to include the effect of $\Delta_{e^2p^6}^{(\pi)}\propto|T_{SD}|^2$, that is formally of $\mathcal{O}(e^2p^6)$, but is not helicity suppressed and behaves

	$(P=\pi)$	$(P=K)$
$\Delta^{(P)}_{e^2p^2}$ (%)	-3.929	-3.786
$\Delta^{(P)}_{e^2p^4}$ (%)	0.053 ± 0.011	0.135 ± 0.011
$\Delta^{(P)}_{e^2p^6}$ (%)	0.073	
Δ_{LL} (%)	0.055	0.055

Table 2: Numerical summary of various electroweak corrections to $R^{(P)}_{e/\mu}$. The uncertainty in $\Delta^{(P)}_{e^2p^4}$ corresponds to the matching procedure.

as $\Delta_{e^2p^6}\sim\alpha/\pi\,(m_P/M_V)^4\,(m_P/m_e)^2$. On the other hand, the usual experimental definition of $R^{(K)}_{e/\mu}$ is not fully inclusive on the radiative mode. It corresponds to including the effect of T_{IB} in $\Delta^{(K)}_{e^2p^2}$ (dominated by soft photons) and excluding altogether the effect of T_{SD}: consequently $c_n^{(\pi)}\neq c_n^{(K)}$.

The expressions of the constants $c^{(P)}_{2,3,4}$ and $\tilde{c}^{(P)}_2$ are shown in Refs. [17, 18] and their numerical values are reported in table 1. Note that to this order in Chiral Perturbation Theory, $R^{(P)}_{e/\mu}$ features both model independent double chiral logarithms (previously neglected) and an a priori unknown low-energy constant. By including the finite loop effects and estimating the low-energy constant via a matching calculation in large-N_C QCD, we thus provide the first complete result of $R^{(P)}_{e/\mu}$ to $\mathcal{O}(e^2p^4)$ in the effective power counting. Most importantly, the matching calculation allows us to further reduce the theoretical uncertainty and put it on more solid ground.

In table 2 we summarize the various electroweak corrections to $R^{(\pi,K)}_{e/\mu}$. Applying these we arrive to our final results:

$$R^{(\pi)}_{e/\mu} = (1.2352\pm 0.0001)\times 10^{-4}\,, \tag{2.7}$$

$$R^{(K)}_{e/\mu} = (2.477\pm 0.001)\times 10^{-5}\,. \tag{2.8}$$

The uncertainty we quote for $R^{(\pi)}_{e/\mu}$ is entirely induced by our matching procedure. However, in the case of $R^{(K)}_{e/\mu}$ we have inflated the nominal uncertainty arising from matching by a factor of four, to account for higher order chiral corrections, that are expected to scale as $\Delta^{(K)}_{e^2p^4}\times m_K^2/(4\pi F)^2$.

3. Discussion

3.1 Comparison to previous theoretical predictions

Our results have to be compared with the previous theoretical predicitions of Refs. [14] and [15], which we report in table 3.

i) $R^{(\pi)}_{e/\mu}$ is in good agreement with both previous results.

ii) There is a discrepancy in $R^{(K)}_{e/\mu}$ that goes well outside the estimated theoretical uncertainties. We have traced back this difference to two problematic aspects of Ref. [15]. The leading log correction Δ_{LL} is included with the wrong sign: this accounts for half of the discrepancy.

	$10^4 \cdot R^{(\pi)}_{e/\mu}$	$10^5 \cdot R^{(K)}_{e/\mu}$
This work	1.2352 ± 0.0001	2.477 ± 0.001
Ref. [14]	1.2352 ± 0.0005	
Ref. [15]	1.2354 ± 0.0002	2.472 ± 0.001

Table 3: Comparison of our result with previous theoretical predictions of $R^{(P)}_{e/\mu}$.

The remaining effect is due to the difference in the next-to-leading order virtual correction, for which Finkemeier finds $\Delta^{(K)}_{e^2p^4} = 0.058\%$. We have serious doubts on the reliability of this number because the hadronic form factors modeled in Ref. [15] do not satisfy the correct QCD short-distance behavior. At high momentum they fall off faster than the QCD requirement, thus leading to a smaller value of $\Delta^{(K)}_{e^2p^4}$ compared to our work.

3.2 Comparison to experiments

3.2.1 $R^{(\pi)}_{e/\mu}$

The three most recent measurements of $R^{(\pi)}_{e/\mu}$ are mutually consistent:

$$
\begin{aligned}
R^{(\pi)}_{e/\mu}|_{\text{Bryman}} &= (1.218 \pm 0.014) \times 10^{-4} \quad \text{Ref. [8]}\,, \\
R^{(\pi)}_{e/\mu}|_{\text{Britton}} &= \left(1.2265 \pm 0.0034_{\text{stat}} \pm 0.0044_{\text{syst}}\right) \times 10^{-4} = (1.227 \pm 0.006) \times 10^{-4} \quad \text{Ref. [9]}\,, \\
R^{(\pi)}_{e/\mu}|_{\text{Czapek}} &= \left(1.2346 \pm 0.0035_{\text{stat}} \pm 0.0036_{\text{syst}}\right) \times 10^{-4} = (1.235 \pm 0.005) \times 10^{-4} \quad \text{Ref. [10]}\,.
\end{aligned}
\tag{3.1}
$$

These measurements are in agreement with the theoretical prediction and they give the PDG average $(1.230 \pm 0.004) \times 10^{-4}$ [25]. Note that it is less accurate than the prediction by a factor of around 40. As it has been indicated previously, the experiments ruled by the PEN Collaboration at the Paul Scherrer Institute [4] and by the PIENU Collaboration at TRIUMF [5] are under way and are expected to improve significantly the uncertainty.

3.2.2 $R^{(K)}_{e/\mu}$

The old measurements of $R^{(K)}_{e/\mu}$ in the seventies are also mutually consistent and in agreement with the Standard Model prediction,

$$
\begin{aligned}
R^{(K)}_{e/\mu}|_{\text{Clark}} &= (2.42 \pm 0.42) \times 10^{-5} \qquad \text{Ref. [11]}\,, \\
R^{(K)}_{e/\mu}|_{\text{Heard}} &= (2.37 \pm 0.17) \times 10^{-5} \qquad \text{Ref. [12]}\,, \\
R^{(K)}_{e/\mu}|_{\text{Heintze}} &= (2.51 \pm 0.15) \times 10^{-5} \qquad \text{Ref. [13]}\,.
\end{aligned}
\tag{3.2}
$$

Using these data the world average reads $(2.45 \pm 0.11) \times 10^{-5}$ [25]. Again it is much less accurate than our theoretical prediction, by a factor of around 100. Recent experiments have been improving significantly the uncertainty:

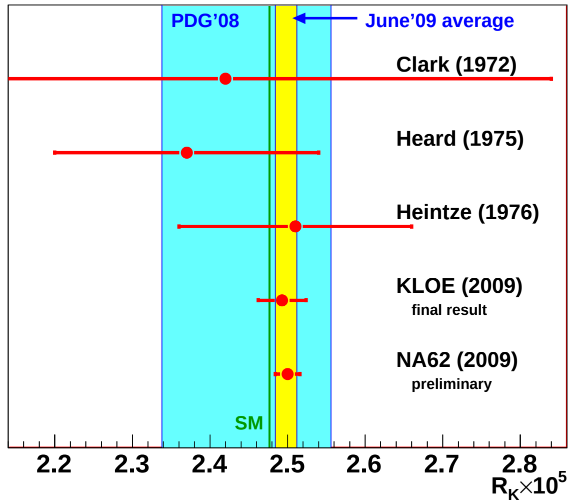

Figure 1: Summary of $R^{(K)}_{e/\mu}$ measurements (picture from [7]).

i) The KLOE Collaboration has recently published the measurement performed at DAFNE [6],

$$R^{(K)}_{e/\mu}|_{\rm KLOE} = \left(2.493 \pm 0.025_{\rm stat} \pm 0.019_{\rm syst}\right) \times 10^{-5} = (2.49 \pm 0.03) \times 10^{-5}\,, \quad (3.3)$$

in agreement with our Standard Model prediction of 2.8. The achieved precision is 1.2%, improving the precision of the former world average by a factor of 4.

ii) On the other hand the NA62 experiment at CERN has also recently announced its preliminary result [7],

$$R^{(K)}_{e/\mu}|_{\rm NA62} = \left(2.500 \pm 0.012_{\rm stat} \pm 0.011_{\rm syst}\right) \times 10^{-5} = (2.500 \pm 0.016) \times 10^{-5}\,. \quad (3.4)$$

The uncertainty is now 0.64%. Note that the whole 2007-08 data sample is supposed to allow pushing the uncertainty down to 0.4%. This result is compatible with the KLOE one and it is also in agreement with the theoretical prediction of 2.8 (at 1.4 σ).

With these new results, and until the final result of NA62 arrives, using a simple weighted mean the new world average reads

$$R^{(K)}_{e/\mu}|_{\rm WA} = (2.498 \pm 0.014) \times 10^{-5}\,, \quad (3.5)$$

which is in agreement with the Standard Model result (at 1.5 σ). A summary of $R^{(K)}_{e/\mu}$ measurements is presented in figure 1.

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