Two-Loop Effective Theory Analysis of $\pi\,(K) \to e\bar{\nu}_e[\gamma]$ Branching Ratios

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We study the ratios $R_{e/\mu}^{(P)} \equiv \Gamma(P \to e\bar{\nu}_e[\gamma])/\Gamma(P \to \mu\bar{\nu}_\mu[\gamma])$ $(P = \pi,\ K)$ in Chiral Perturbation Theory to order e^2p^4. We complement the two-loop effective theory results with a matching calculation of the counterterm, finding $R_{e/\mu}^{(\pi)} = (1.2352 \pm 0.0001) \times 10^{-4}$ and $R_{e/\mu}^{(K)} = (2.477 \pm 0.001) \times 10^{-5}$.

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Introduction.—The ratio $R_{e/\mu}^{(P)} \equiv \Gamma(P \to e\bar{\nu}_e[\gamma])/\Gamma(P \to \mu\bar{\nu}_\mu[\gamma])$ $(P = \pi, K)$ is helicity suppressed in the Standard Model (SM), due to the $V - A$ structure of charged current couplings. It is therefore a sensitive probe of all SM extensions that induce pseudoscalar currents and nonuniversal corrections to the lepton couplings [1], such as the minimal supersymmetric SM [2]. Effects from weak-scale new physics are expected in the range $(\Delta R_{e/\mu})/R_{e/\mu} \sim 10^{-4}$–$10^{-2}$, and there is a realistic chance to detect or constrain them because: (i) ongoing experimental searches plan to reach a fractional uncertainty of $(\Delta R_{e/\mu}^{(\pi)})/R_{e/\mu}^{(\pi)} \lesssim 5 \times 10^{-4}$ [3] and $(\Delta R_{e/\mu}^{(K)})/R_{e/\mu}^{(K)} \lesssim 3 \times 10^{-3}$ [4], which represent, respectively, a factor of 5 and 10 improvement over current errors [5]. (ii) The SM theoretical uncertainty can be pushed below this level, since to a first approximation the strong interaction dynamics cancels out in the ratio $R_{e/\mu}$ and hadronic structure dependence appears only through electroweak corrections. Indeed, the most recent theoretical predictions read $R_{e/\mu}^{(\pi)} = (1.2352 \pm 0.0005) \times 10^{-4}$ [6], $R_{e/\mu}^{(\pi)} = (1.2354 \pm 0.0002) \times 10^{-4}$ [7], and $R_{e/\mu}^{(K)} = (2.472 \pm 0.001) \times 10^{-5}$ [7]. The authors of Ref. [6] provide a general parameterization of the hadronic effects and estimate the induced uncertainty via dimensional analysis. On the other hand, in Ref. [7], the hadronic component is calculated by modeling the low- and intermediate-momentum region of the loops involving virtual photons.

With the aim to improve the existing theoretical status, we have analyzed $R_{e/\mu}$ within Chiral Perturbation Theory (ChPT), the low-energy effective field theory (EFT) of QCD. The key feature of this framework is that it provides a controlled expansion of the amplitudes in terms of the masses of pseudoscalar mesons and charged leptons ($p \sim m_{\pi,K,\ell}/\Lambda_\chi$, with $\Lambda_\chi \sim 4\pi F_\pi \sim 1.2$ GeV), and the electromagnetic coupling (e). Electromagnetic corrections to (semi)-leptonic decays of K and π have been worked out to $O(e^2p^2)$ [8,9], but had never been pushed to $O(e^2p^4)$, as required for $R_{e/\mu}$. In this Letter, we report the results of our analysis of $R_{e/\mu}$ to $O(e^2p^4)$, deferring the full details to a separate publication [10]. To the order we work, $R_{e/\mu}$ features both model independent double chiral logarithms (previously neglected) and an *a priori* unknown low-energy coupling (LEC), which we estimate by means of a matching calculation in large-N_C QCD. The inclusion of both effects allows us to further reduce the theoretical uncertainty and to put its estimate on more solid ground.

Within the chiral power counting, $R_{e/\mu}$ is written as

$$R_{e/\mu}^{(P)} = R_{e/\mu}^{(0),(P)}[1 + \Delta_{e^2p^2}^{(P)} + \Delta_{e^2p^4}^{(P)} + \Delta_{e^2p^6}^{(P)} + \ldots] \quad (1)$$

$$R_{e/\mu}^{(0),(P)} = \frac{m_e^2}{m_\mu^2}\left(\frac{m_P^2 - m_e^2}{m_P^2 - m_\mu^2}\right)^2. \quad (2)$$

The leading electromagnetic correction $\Delta_{e^2p^2}^{(P)}$ corresponds to the pointlike approximation for pion and kaon, and its expression is well known [6,11]. Neglecting terms of order $(m_e/m_\rho)^2$, the most general parameterization of the next-to-leading order (NLO) ChPT contribution can be written in the form

$$\Delta_{e^2p^4}^{(P)} = \frac{\alpha}{\pi}\frac{m_\mu^2}{m_\rho^2}\left(c_2^{(P)}\log\frac{m_\rho^2}{m_\mu^2} + c_3^{(P)} + c_4^{(P)}(m_\mu/m_P)\right) + \frac{\alpha}{\pi}\frac{m_P^2}{m_\rho^2}\tilde{c}_2^{(P)}\log\frac{m_\mu^2}{m_e^2}, \quad (3)$$

which highlights the dependence on lepton masses. The dimensionless constants $c_{2,3}^{(P)}$ do not depend on the lepton mass but depend logarithmically on hadronic masses, while $c_4^{(P)}(m_\mu/m_P) \to 0$ as $m_\mu \to 0$. (Note that our $c_{2,3}^{(\pi)}$ do not coincide with $C_{2,3}$ of Ref. [6] because their C_3 is not constrained to be m_ℓ-independent.) Finally, depending on the treatment of real photon emission, one has to include in $R_{e/\mu}$ terms arising from the structure dependent contribution to $P \to e\bar{\nu}_e\gamma$ [12] that are formally of $O(e^2p^6)$, but are not helicity suppressed and behave as $\Delta_{e^2p^6} \sim \alpha/\pi(m_P/m_\rho)^4(m_P/m_e)^2$.

The calculation.—In order to calculate the various coefficients $c_i^{(P)}$ within ChPT to $O(e^2p^4)$, one has to consider (i) two-loop graphs with vertices from the lowest order effective Lagrangian [$O(p^2)$]; (ii) one-loop graphs with one insertion from the NLO Lagrangian [13] [$O(p^4)$];

(iii) tree-level diagrams with insertion of a local counterterm of $O(e^2p^4)$. In Fig. 1, we show all the relevant one- and two-loop 1PI topologies contributing to $R_{e/\mu}$. Note that all diagrams in which the virtual photon does not connect to the charged lepton line have a trivial dependence on the lepton mass and drop when taking the ratio of e and μ rates. We work in Feynman gauge and use dimensional regularization to deal with ultraviolet (UV) divergences.

By suitably grouping the 1PI graphs of Fig. 1 with external leg corrections, it is possible to show [10] that the effect of the $O(e^2p^4)$ diagrams amounts to: (i) a renormalization of the meson mass m_P and decay constant F_P in the one-loop result $\Delta^{(P)}_{e^2p^2}$; (ii) a genuine shift to the invariant amplitude $T_\ell \equiv T(P^+(p) \to \ell^+(p_\ell)\nu_\ell(p_\nu))$. This correction can be expressed as the convolution of a known kernel with the vertex function $\mathcal{T}_{\mu\nu} = 1/(\sqrt{2}F) \times \int dx e^{iqx+iWy}\langle 0|T[J^{\rm EM}_\mu(x)(V_\nu - A_\nu)(y)]|\pi^+(p)\rangle$ [with $V_\mu(A_\mu) = \bar{u}\gamma_\mu(\gamma_5)d$], once the Born term has been subtracted from the latter. Explicitly, in the case of pion decay, one has ($W = p - q$, $\epsilon_{0123} = +1$)

$$\delta T_\ell^{e^2p^4} = 2G_F V_{ud}^* e^2 F \int \frac{d^dq}{(2\pi)^d} \frac{\bar{u}_L(p_\nu)\gamma^\nu[-(\not{p}_l - \not{q}) + m_\ell]\gamma^\mu v(p_\ell)}{[q^2 - 2q\cdot p_\ell + i\epsilon][q^2 - m_\gamma^2 + i\epsilon]}\mathcal{T}_{\mu\nu}(p,q) \tag{4}$$

$$\mathcal{T}^{\mu\nu}(p,q) = iV_1(q^2,W^2)\epsilon^{\mu\nu\alpha\beta}q_\alpha p_\beta - A_1(q^2,W^2)(q\cdot p g^{\mu\nu} - p^\mu q^\nu) - [A_2(q^2,W^2) - A_1(q^2,W^2)](q^2g^{\mu\nu} - q^\mu q^\nu) + \left[\frac{(2p-q)^\mu(p-q)^\nu}{2p\cdot q - q^2} - \frac{q^\mu(p-q)^\nu}{q^2}\right][F_V^{\pi\pi}(q^2) - 1]. \tag{5}$$

To the order we work, the form factors $V_1(q^2,W^2)$, $A_i(q^2,W^2)$, and $F_V^{\pi\pi}(q^2)$ have to be evaluated to $O(p^4)$ in ChPT in d-dimensions. Their expressions are well known for $d = 4$ [12] and have been generalized to any d [10]. So the relevant $O(e^2p^4)$ amplitude is obtained by calculating a set of one-loop diagrams with effective local (V_1 and A_1) and nonlocal (A_2 and $F_V^{\pi\pi}$) $O(p^4)$ vertices. The final result can be expressed in terms of one-dimensional integrals [10].

While $c^{(P)}_{2,4}$ and $\tilde{c}^{(P)}_2$ are parameter-free predictions of ChPT (they depend only on $m_{\pi,K}$, F_π, and the LECs $L_{9,10}$ determined in other processes [13]), $c_3^{(P)}$ contains an ultraviolet (UV) divergence, indicating the need to introduce in the effective theory a local operator of $O(e^2p^4)$, with an associated LEC. The physical origin of the UV divergence is clear: when calculating $\delta T_\ell^{e^2p^4}$ in the EFT approach, we use the $O(p^4)$ ChPT representation of the form factors appearing in Eq. (5) ($\mathcal{T}_{\mu\nu} \to \mathcal{T}_{\mu\nu}^{\rm ChPT}$). While this representation is valid at scales below m_ρ (and generates the correct single- and double-logs upon integration in d^dq), it leads to the incorrect UV behavior of the integrand in Eq. (4), which is instead dictated by the Operator Product Expansion (OPE) for the $\langle VVP\rangle$ and $\langle VAP\rangle$ correlators. So in order to estimate the finite local contribution (dominated by the UV region), we need a QCD representation of the correlators valid for momenta beyond the chiral regime ($\mathcal{T}_{\mu\nu} \to \mathcal{T}_{\mu\nu}^{\rm QCD}$). This program is feasible only within an approximation scheme to QCD. We have used a truncated version of large-N_C QCD, in which the correlators are approximated by meromorphic functions, representing the exchange of a *finite* number of narrow resonances, whose couplings are fixed by requiring that the vertex functions $\langle\pi|VA|0\rangle$ and $\langle\pi|VV|0\rangle$ obey the leading and next-to-leading OPE behavior at large q [14]. This procedure allows us to obtain a simple analytic form for the local coupling [see Eq. (10)].

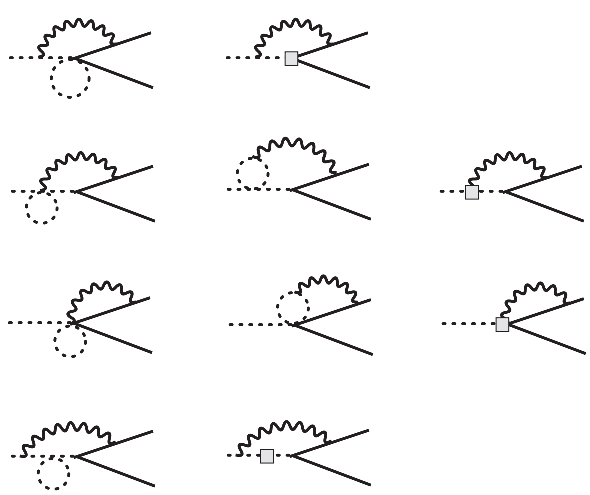

FIG. 1. One- and two-loop 1PI topologies contributing to $R_{e/\mu}$ to order e^2p^4. Dashed lines represent pseudoscalar mesons, solid lines fermions and wavy lines photons. Shaded squares indicate vertices from the $O(p^4)$ effective Lagrangian.

Results.—The results for $c^{(P)}_{2,3,4}$ and $\tilde{c}^{(P)}_2$ depend on the definition of the inclusive rate $\Gamma(P \to \ell\bar{\nu}_\ell[\gamma])$. The radiative amplitude is the sum of the inner bremsstrahlung component ($T_{\rm IB}$) of $O(ep)$ and a structure dependent component ($T_{\rm SD}$) of $O(ep^3)$ [12]. The experimental definition of $R^{(\pi)}_{e/\mu}$ is fully inclusive on the radiative mode, so that $\Delta^{(\pi)}_{e^2p^4}$ receives a contribution from the interference of $T_{\rm IB}$ and $T_{\rm SD}$, and one also has to include the effect of $\Delta^{(\pi)}_{e^2p^6} \propto |T_{\rm SD}|^2$. The usual experimental definition of $R^{(K)}_{e/\mu}$ corresponds to including the effect of $T_{\rm IB}$ in $\Delta^{(K)}_{e^2p^2}$ (dominated by soft photons) and excluding altogether the effect of $T_{\rm SD}$; consequently, $c_n^{(\pi)} \neq c_n^{(K)}$.

Results for $R^{(\pi)}_{e/\mu}$.—Defining $\bar{L}_9 \equiv (4\pi)^2 L_9^r(\mu)$, $\ell_P \equiv \log(m_P^2/\mu^2)$ (μ is the chiral renormalization scale), $\gamma \equiv A_1(0,0)/V_1(0,0)$, $z_\ell \equiv (m_\ell/m_\pi)^2$, we find

$$c_2^{(\pi)} = \frac{2}{3} m_\rho^2 \langle r^2\rangle_V^{(\pi)} + 3(1-\gamma)\frac{m_\rho^2}{(4\pi F)^2} \qquad \tilde{c}_2^{(\pi)} = 0 \quad (6)$$

$$\begin{aligned} c_3^{(\pi)} = -\frac{m_\rho^2}{(4\pi F)^2}\Big[&\frac{31}{24} - \gamma + 4\bar{L}_9 + \Big(\frac{23}{36} - 2\bar{L}_9 + \frac{1}{12}\ell_K\Big)\ell_\pi \\ &+ \frac{5}{12}\ell_\pi^2 + \frac{5}{18}\ell_K + \frac{1}{8}\ell_K^2 + \Big(\frac{5}{3} - \frac{2}{3}\gamma\Big)\log\frac{m_\rho^2}{m_\pi^2} \\ &+ \Big(2 + 2\kappa^{(\pi)} - \frac{7}{3}\gamma\Big)\log\frac{m_\rho^2}{\mu^2} + K^{(\pi)}(0)\Big] + c_3^{CT}(\mu) \quad (7) \end{aligned}$$

$$\begin{aligned} c_4^{(\pi)}(m_\ell) = -\frac{m_\rho^2}{(4\pi F)^2}\Big\{&\frac{z_\ell}{3(1-z_\ell)^2} \\ &\times [(4(1-z_\ell) + (9-5z_\ell)\log z_\ell) \\ &+ 2\gamma(1 - z_\ell + z_\ell \log z_\ell)] \\ &+ \Big(\kappa^{(\pi)} + \frac{1}{3}\Big)\frac{z_\ell}{2(1-z_\ell)}\log z_\ell \\ &+ K^{(\pi)}(z_\ell) - K^{(\pi)}(0)\Big\} \quad (8) \end{aligned}$$

where $\kappa^{(\pi)}$ is related to the $O(p^4)$ pion charge radius by

$$\kappa^{(\pi)} \equiv 4\bar{L}_9 - \frac{1}{6}\ell_K - \frac{1}{3}\ell_\pi - \frac{1}{2} = \frac{(4\pi F)^2}{3}\langle r^2\rangle_V^{(\pi)}. \quad (9)$$

The function $K^{(\pi)}(z_\ell)$, whose expression will be given in Ref. [10], does not contain any large logarithms and gives a small fractional contribution to $c_{3,4}^{(\pi)}$.

As anticipated, $c_2^{(\pi)}$ is a parameter-free prediction of ChPT. Moreover, we find $\tilde{c}_2^{(\pi)} = 0$, as expected due to a cancellation of real- and virtual-photon effects [15]. Finally, $c_3^{(\pi)}$ encodes calculable chiral corrections [as does $c_4(m_\ell)$] and a local counterterm $c_3^{CT}(\mu)$, for which our matching procedure [10] gives ($z_A \equiv m_{a_1}/m_\rho$)

$$\begin{aligned} c_3^{CT}(\mu) = &-\frac{19 m_\rho^2}{9(4\pi F)^2} + \Big(\frac{4 m_\rho^2}{3(4\pi F)^2} + \frac{7 + 11 z_A^2}{6 z_A^2}\Big)\log\frac{m_\rho^2}{\mu^2} \\ &+ \frac{37 - 31 z_A^2 + 17 z_A^4 - 11 z_A^6}{36 z_A^2 (1 - z_A^2)^2} \\ &- \frac{7 - 5 z_A^2 - z_A^4 + z_A^6}{3 z_A^2(-1 + z_A^2)^3}\log z_A. \quad (10) \end{aligned}$$

Numerically, using $z_A = \sqrt{2}$, we find $c_3^{CT}(m_\rho) = -1.61$, implying that the counterterm induces a subleading correction to c_3 (see Table I). The scale dependence of $c_3^{CT}(\mu)$ partially cancels the scale dependence of the chiral loops (our procedure captures all the "single-log" scale dependence). Taking a very conservative attitude, we assign to c_3 an uncertainty equal to 100% of the local contribution ($|\Delta c_3| \sim 1.6$) plus the effect of residual renormalization scale dependence, obtained by varying the scale μ in the range $0.5 \to 1$ GeV ($|\Delta c_3| \sim 0.7$), leading to $\Delta c_3^{(\pi,K)} = \pm 2.3$. Full numerical values of $c_{2,3,4}^{(\pi)}$ are reported in Table I, with uncertainties due to matching procedure and input parameters (L_9 and γ [16]).

As a check on our calculation, we have verified that if we neglect c_3^{CT} and pure two-loop effects, and if we use $L_9 = F^2/(2m_\rho^2)$ (vector meson dominance), our results for $c_{2,3,4}^{(\pi)}$ are fully consistent with previous analyses of the leading structure dependent corrections based on current algebra [6,17]. Moreover, our numerical value of $\Delta^{(\pi)}_{e^2p^4}$ reported in Table II is very close to the corresponding result in Ref. [6], $\Delta^{(\pi)}_{e^2p^4} = (0.054 \pm 0.044) \times 10^{-2}$.

For completeness, we report here the contribution to $\Delta^{(\pi)}_{e^2p^6}$ induced by structure dependent radiation:

$$\begin{aligned} \Delta^{(\pi)}_{e^2p^6} = \frac{\alpha}{2\pi}\frac{m_\pi^4}{(4\pi F)^4}(1+\gamma^2)\Big[&\frac{1}{30 z_e} - \frac{11}{60} + \frac{z_e}{20(1-z_e)^2} \\ &\times (12 - 3z_e - 10 z_e^2 + z_e^3 + 20 z_e \log z_e)\Big]. \quad (11) \end{aligned}$$

Results for $R^{(K)}_{e/\mu}$.—In this case, we have

$$c_2^{(K)} = \frac{2}{3} m_\rho^2 \langle r^2\rangle_V^{(K)} + \frac{4}{3}\Big(1 - \frac{7}{4}\gamma\Big)\frac{m_\rho^2}{(4\pi F)^2} \quad (12)$$

$$\tilde{c}_2^{(K)} = \frac{1}{3}(1-\gamma)\frac{m_\rho^2}{(4\pi F)^2} \quad (13)$$

where $\langle r^2\rangle_V^{(K)}$ is the $O(p^4)$ kaon charge radius. $c_3^{(K)}$ is obtained from $c_3^{(\pi)}$ by replacing $31/24 - \gamma \to -7/72 - 13/9\gamma$, by dropping the term proportional to $\log m_\rho^2/m_\pi^2$, and by interchanging everywhere else the label π with K (masses, $\ell_\pi \to \ell_K$, etc.). $c_4^{(K)}$ is obtained from $c_4^{(\pi)}$ by keeping only the fourth and fifth lines of Eq. (8) and interchanging the labels π and K. The numerical values of $c_{2,3,4}^{(K)}$ and $\tilde{c}_2^{(K)}$ are reported in Table I.

Resumming leading logarithms.—At the level of uncertainty considered, one needs to include higher order long distance corrections to the leading contribution $\Delta_{e^2p^2} \sim -3\alpha/\pi \log m_\mu/m_e \sim -3.7\%$. The leading logarithms can

TABLE I. Numerical values of the coefficients $c_n^{(P)}$ of Eq. (3) ($P = \pi, K$). The uncertainties correspond to the input values $L_9^r(\mu = m_\rho) = (6.9 \pm 0.7) \times 10^{-3}$, $\gamma = 0.465 \pm 0.005$ [16], and to the matching procedure (m), affecting only $c_3^{(P)}$.

	$(P = \pi)$	$(P = K)$
$\tilde{c}_2^{(P)}$	0	$(7.84 \pm 0.07_\gamma) \times 10^{-2}$
$c_2^{(P)}$	$5.2 \pm 0.4_{L_9} \pm 0.01_\gamma$	$4.3 \pm 0.4_{L_9} \pm 0.01_\gamma$
$c_3^{(P)}$	$-10.5 \pm 2.3_m \pm 0.53_{L_9}$	$-4.73 \pm 2.3_m \pm 0.28_{L_9}$
$c_4^{(P)}(m_\mu)$	$1.69 \pm 0.07_{L_9}$	$0.22 \pm 0.01_{L_9}$

TABLE II. Numerical summary of various electroweak corrections to $R_{e/\mu}^{(\pi,K)}$.

	$(P=\pi)$	$(P=K)$
$\Delta_{e^2p^2}^{(P)}$ (%)	-3.929	-3.786
$\Delta_{e^2p^4}^{(P)}$ (%)	0.053 ± 0.011	0.135 ± 0.011
$\Delta_{e^2p^6}^{(P)}$ (%)	0.073	
Δ_{LL} (%)	0.055	0.055

be summed via the renormalization group and their effect amounts to multiplying $R_{e/\mu}^{(P)}$ by [6]

$$1 + \Delta_{LL} = \frac{(1 - \frac{2}{3}\frac{\alpha}{\pi}\log\frac{m_\mu}{m_e})^{9/2}}{1 - \frac{3\alpha}{\pi}\log\frac{m_\mu}{m_e}} = 1.00055. \qquad (14)$$

Conclusions.—In Table II, we summarize the various corrections to $R_{e/\mu}^{(\pi,K)}$, which lead to our final results:

$$R_{e/\mu}^{(\pi)} = (1.2352 \pm 0.0001) \times 10^{-4} \qquad (15)$$

$$R_{e/\mu}^{(K)} = (2.477 \pm 0.001) \times 10^{-5}. \qquad (16)$$

In the case of $R_{e/\mu}^{(K)}$, we have inflated the nominal uncertainty arising from matching by a factor of 4, to account for higher order chiral corrections of expected size $\Delta_{e^2p^4} m_K^2/(4\pi F)^2$. The analogous corrections to $R_{e/\mu}^{(\pi)}$ scale like $\Delta_{e^2p^4} m_\pi^2/(4\pi F)^2$ and are negligible. Our results have to be compared with the ones of Refs. [6,7] reported in the introduction. While $R_{e/\mu}^{(\pi)}$ is in good agreement with both previous results, there is a discrepancy in $R_{e/\mu}^{(K)}$ that goes well outside the estimated theoretical uncertainties. We have traced back this difference to the following problems in Ref. [7]: (i) the leading log correction Δ_{LL} is included with the wrong sign (this accounts for half of the discrepancy); (ii) the NLO virtual correction $\Delta_{e^2p^4}^{(K)} = 0.058\%$ is not reliable because the hadronic form factors modeled in Ref. [7] do not satisfy the QCD short-distance behavior.

In conclusion, by performing an analysis to $O(e^2p^4)$ in ChPT, we have improved the reliability of both the central value and the uncertainty of the ratios $R_{e/\mu}^{(\pi,K)}$. Our final result for $R_{e/\mu}^{(\pi)}$ is consistent with the previous literature, while we find a discrepancy in $R_{e/\mu}^{(K)}$, which we have traced back to inconsistencies in the analysis of Ref. [7]. Our results provide a clean basis to detect or constrain nonstandard physics in these channels by comparison with upcoming measurements.

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