LAUR-07-3194

The Standard Model prediction for $R_{e/\mu}^{(\pi,K)}$

Vincenzo Cirigliano[1] and Ignasi Rosell[2]
[1] Theoretical Division, Los Alamos National Laboratory, Los Alamos NM 87544, USA
[2] Departamento de Ciencias Físicas, Matemáticas y de la Computación,
Universidad CEU Cardenal Herrera, San Bartolomé 55, E-46115 Alfara del Patriarca, València, Spain

We study the ratios $R_{e/\mu}^{(P)} \equiv \Gamma(P \to e\bar{\nu}_e[\gamma])/\Gamma(P \to \mu\bar{\nu}_\mu[\gamma])$ ($P = \pi, K$) in Chiral Perturbation Theory to order e^2p^4. We complement the two-loop effective theory results with a matching calculation of the counterterm, finding $R_{e/\mu}^{(\pi)} = (1.2352 \pm 0.0001) \times 10^{-4}$ and $R_{e/\mu}^{(K)} = (2.477 \pm 0.001) \times 10^{-5}$.

arXiv:0707.3439v1 [hep-ph] 23 Jul 2007

Introduction - The ratio $R_{e/\mu}^{(P)} \equiv \Gamma(P \to e\bar{\nu}_e[\gamma])/\Gamma(P \to \mu\bar{\nu}_\mu[\gamma])$ ($P = \pi, K$) is helicity-suppressed in the Standard Model (SM), due to the $V-A$ structure of charged current couplings. It is therefore a sensitive probe of all SM extensions that induce pseudoscalar currents and non-universal corrections to the lepton couplings [1], such as the minimal supersymmetric SM [2]. Effects from weak-scale new physics are expected in the range $(\Delta R_{e/\mu})/R_{e/\mu} \sim 10^{-4} - 10^{-2}$ and there is a realistic chance to detect or constrain them because: (i) ongoing experimental searches plan to reach a fractional uncertainty of $(\Delta R_{e/\mu}^{(\pi)})/R_{e/\mu}^{(\pi)} \lesssim 5 \times 10^{-4}$ [3] and $(\Delta R_{e/\mu}^{(K)})/R_{e/\mu}^{(K)} \lesssim 3 \times 10^{-3}$ [4], which represent respectively a factor of 5 and 10 improvement over current errors [5]. (ii) The SM theoretical uncertainty can be pushed below this level, since to a first approximation the strong interaction dynamics cancels out in the ratio $R_{e/\mu}$ and hadronic structure dependence appears only through electroweak corrections. Indeed, the most recent theoretical predictions read $R_{e/\mu}^{(\pi)} = (1.2352 \pm 0.0005) \times 10^{-4}$ [6], $R_{e/\mu}^{(\pi)} = (1.2354 \pm 0.0002) \times 10^{-4}$ [7], and $R_{e/\mu}^{(K)} = (2.472 \pm 0.001) \times 10^{-5}$ [7]. The authors of Ref. [6] provide a general parameterization of the hadronic effects and estimate the induced uncertainty via dimensional analysis. On the other hand, in Ref. [7] the hadronic component is calculated by modeling the low- and intermediate-momentum region of the loops involving virtual photons.

With the aim to improve the existing theoretical status, we have analyzed $R_{e/\mu}$ within Chiral Perturbation Theory (ChPT), the low-energy effective field theory (EFT) of QCD. The key feature of this framework is that it provides a controlled expansion of the amplitudes in terms of the masses of pseudoscalar mesons and charged leptons ($p \sim m_{\pi,K,\ell}/\Lambda_\chi$, with $\Lambda_\chi \sim 4\pi F_\pi \sim 1.2\,\text{GeV}$), and the electromagnetic coupling (e). Electromagnetic corrections to (semi)-leptonic decays of K and π have been worked out to $O(e^2p^2)$ [8, 9], but had never been pushed to $O(e^2p^4)$, as required for $R_{e/\mu}$. In this letter we report the results of our analysis of $R_{e/\mu}$ to $O(e^2p^4)$, deferring the full details to a separate publication [10]. To the order we work, $R_{e/\mu}$ features both model independent double chiral logarithms (previously neglected) and an a priori unknown low-energy coupling (LEC), which we estimate by means of a matching calculation in large-N_C QCD. The inclusion of both effects allows us to further reduce the theoretical uncertainty and to put its estimate on more solid ground.

Within the chiral power counting, $R_{e/\mu}$ is written as:

$$R_{e/\mu}^{(P)} = R_{e/\mu}^{(0),(P)} \left[1 + \Delta_{e^2p^2}^{(P)} + \Delta_{e^2p^4}^{(P)} + \Delta_{e^2p^6}^{(P)} + ... \right] \quad (1)$$

$$R_{e/\mu}^{(0),(P)} = \frac{m_e^2}{m_\mu^2} \left(\frac{m_P^2 - m_e^2}{m_P^2 - m_\mu^2} \right)^2 . \quad (2)$$

The leading electromagnetic correction $\Delta_{e^2p^2}^{(P)}$ corresponds to the point-like approximation for pion and kaon, and its expression is well known [6, 11]. Neglecting terms of order $(m_e/m_\rho)^2$, the most general parameterization of the NLO ChPT contribution can be written in the form

$$\Delta_{e^2p^4}^{(P)} = \frac{\alpha}{\pi} \frac{m_\mu^2}{m_\rho^2} \left(c_2^{(P)} \log \frac{m_\rho^2}{m_\mu^2} + c_3^{(P)} + c_4^{(P)}(m_\mu/m_P) \right) + \frac{\alpha}{\pi} \frac{m_P^2}{m_\rho^2} \tilde{c}_2^{(P)} \log \frac{m_\mu^2}{m_e^2} , \quad (3)$$

which highlights the dependence on lepton masses. The dimensionless constants $c_{2,3}^{(P)}$ do not depend on the lepton mass but depend logarithmically on hadronic masses, while $c_4^{(P)}(m_\mu/m_P) \to 0$ as $m_\mu \to 0$. (Note that our $c_{2,3}^{(\pi)}$ do not coincide with $C_{2,3}$ of Ref. [6], because their C_3 is not constrained to be m_ℓ-independent.) Finally, depending on the treatment of real photon emission, one has to include in $R_{e/\mu}$ terms arising from the structure dependent contribution to $P \to e\bar{\nu}_e\gamma$ [12], that are formally of $O(e^2p^6)$, but are not helicity suppressed and behave as

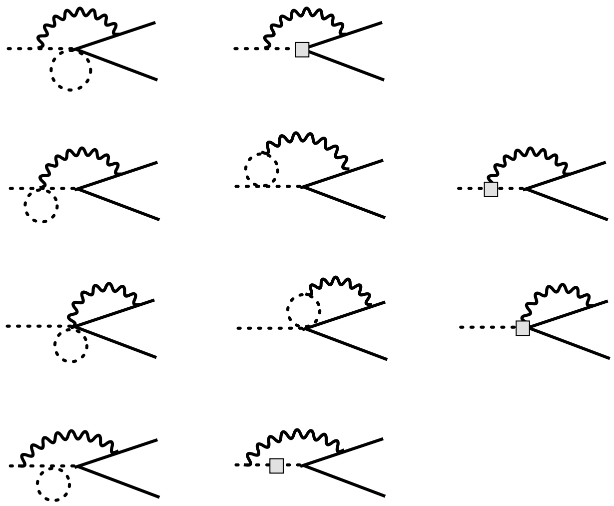

FIG. 1: One- and two-loop 1PI topologies contributing to $R_{e/\mu}$ to order e^2p^4. Dashed lines represent pseudoscalar mesons, solid lines fermions an wavy lines photons. Shaded squares indicate vertices from the $O(p^4)$ effective lagrangian.

$\Delta_{e^2p^6} \sim \alpha/\pi\,(m_P/m_\rho)^4\,(m_P/m_e)^2$.

The calculation - In order to calculate the various coefficients $c_i^{(P)}$ within ChPT to $O(e^2p^4)$, one has to consider (i) two-loop graphs with vertices from the lowest order effective lagrangian ($O(p^2)$); (ii) one-loop graphs with one insertion from the NLO lagrangian [13] ($O(p^4)$); (iii) tree-level diagrams with insertion of a local counterterm of $O(e^2p^4)$. In Fig. 1 we show all the relevant one- and two-loop 1PI topologies contributing to $R_{e/\mu}$. Note that all diagrams in which the virtual photon does not connect to the charged lepton line have a trivial dependence on the lepton mass and drop when taking the ratio of e and μ rates. We work in Feynman gauge and use dimensional regularization to deal with ultraviolet (UV) divergences.

By suitably grouping the 1PI graphs of Fig. 1 with external leg corrections, it is possible to show [10] that the effect of the $O(e^2p^4)$ diagrams amounts to: (i) a renormalization of the meson mass m_P and decay constant F_P in the one-loop result $\Delta_{e^2p^2}^{(P)}$; (ii) a genuine shift to the invariant amplitude $T_\ell \equiv T(P^+(p) \to \ell^+(p_\ell)\nu_\ell(p_\nu))$. This correction can be expressed as the convolution of a known kernel with the vertex function $\mathcal{T}_{\mu\nu} = 1/(\sqrt{2}F)\int dx\; e^{iqx+iWy}\;\langle 0|T(J_\mu^{EM}(x)\,(V_\nu - A_\nu)(y)|\pi^+(p)\rangle$ (with $V_\mu(A_\mu) = \bar u\gamma_\mu(\gamma_5)d$), once the Born term has been subtracted from the latter. Explicitly, in the case of pion decay one has ($W = p - q$, $\epsilon_{0123} = +1$)

$$\delta T_\ell^{e^2p^4} = 2G_F V_{ud}^* e^2 F \int \frac{d^dq}{(2\pi)^d}\,\frac{\bar u_L(p_\nu)\gamma^\nu\left[-(\not p_\ell - \not q) + m_\ell\right]\gamma^\mu v(p_\ell)}{\left[q^2 - 2q\cdot p_\ell + i\epsilon\right]\left[q^2 - m_\gamma^2 + i\epsilon\right]}\;\mathcal{T}_{\mu\nu}(p,q) \tag{4}$$

$$\mathcal{T}^{\mu\nu}(p,q) = iV_1(q^2,W^2)\,\epsilon^{\mu\nu\alpha\beta}q_\alpha p_\beta - A_1(q^2,W^2)\,\left(q\cdot p g^{\mu\nu} - p^\mu q^\nu\right) - \left(A_2(q^2,W^2) - A_1(q^2,W^2)\right)\left(q^2 g^{\mu\nu} - q^\mu q^\nu\right) + \left[\frac{(2p-q)^\mu(p-q)^\nu}{2p\cdot q - q^2} - \frac{q^\mu(p-q)^\nu}{q^2}\right]\left(F_V^{\pi\pi}(q^2) - 1\right)\;. \tag{5}$$

To the order we work, the form factors $V_1(q^2,W^2)$, $A_i(q^2,W^2)$ and $F_V^{\pi\pi}(q^2)$ have to be evaluated to $O(p^4)$ in ChPT in d-dimensions. Their expressions are well known for $d = 4$ [12] and have been generalized to any d [10]. So the relevant $O(e^2p^4)$ amplitude is obtained by calculating a set of one-loop diagrams with effective local (V_1 and A_1) and non-local (A_2 and $F_V^{\pi\pi}$) $O(p^4)$ vertices. The final result can be expressed in terms of one-dimensional integrals [10].

While $c_{2,4}^{(P)}$ and $\tilde c_2^{(P)}$ are parameter-free predictions of ChPT (they depend only on $m_{\pi,K}$, F_π, and the LECs $L_{9,10}$ determined in other processes [13]), $c_3^{(P)}$ contains an ultraviolet (UV) divergence, indicating the need to introduce in the effective theory a local operator of $O(e^2p^4)$, with an associated LEC. The physical origin of the UV divergence is clear: when calculating $\delta T_\ell^{e^2p^4}$ in the EFT approach, we use the $O(p^4)$ ChPT representation of the form factors appearing in Eq. 5 ($\mathcal{T}_{\mu\nu} \to \mathcal{T}_{\mu\nu}^{\rm ChPT}$). While this representation is valid at scales below m_ρ (and generates the correct single- and double-logs upon integration in d^dq) it leads to the incorrect UV behavior of the integrand in Eq. 4, which is instead dictated by the Operator Product Expansion (OPE) for the $\langle VVP\rangle$ and $\langle VAP\rangle$ correlators. So in order to estimate the finite local contribution (dominated by the UV region) we need a QCD representation of the correlators valid for momenta beyond the chiral regime ($\mathcal{T}_{\mu\nu} \to \mathcal{T}_{\mu\nu}^{\rm QCD}$) . This program is feasible only within an approximation scheme to QCD. We have used a truncated version of large-N_C QCD, in which the correlators are approximated by meromorphic functions, representing the exchange of a *finite* number of narrow resonances, whose couplings are fixed by requiring that the vertex functions $\langle\pi|VA|0\rangle$ and $\langle\pi|VV|0\rangle$ obey the leading and next-to-leading OPE behavior at large q [14]. This procedure allows us to obtain a simple analytic form for the local coupling (see Eq. 10).

Results - The results for $c_{2,3,4}^{(P)}$ and $\tilde c_2^{(P)}$ depend on the definition of the inclusive rate $\Gamma(P \to \ell\bar\nu_\ell[\gamma])$. The radiative amplitude is the sum of the inner bremsstrahlung component (T_{IB}) of $O(ep)$ and a structure dependent component (T_{SD}) of $O(ep^3)$ [12]. The experimental definition of $R^{(\pi)}_{e/\mu}$ is fully inclusive on the radiative mode, so that $\Delta^{(\pi)}_{e^2p^4}$ receives a contribution from the interference of T_{IB} and T_{SD}, and one also has to include the effect of $\Delta^{(\pi)}_{e^2p^6} \propto |T_{SD}|^2$. The usual experimental definition of $R^{(K)}_{e/\mu}$ corresponds to including the effect of T_{IB} in $\Delta^{(K)}_{e^2p^2}$ (dominated by soft photons) and excluding altogether the effect of T_{SD}: consequently $c_n^{(\pi)} \neq c_n^{(K)}$.

Results for $R^{(\pi)}_{e/\mu}$ - Defining $\bar{L}_9 \equiv (4\pi)^2 L_9^r(\mu)$, $\ell_P \equiv \log(m_P^2/\mu^2)$ (μ is the chiral renormalization scale), $\gamma \equiv A_1(0,0)/V_1(0,0)$, $z_\ell \equiv (m_\ell/m_\pi)^2$, we find:

$$c_2^{(\pi)} = \frac{2}{3}\, m_\rho^2 \, \langle r^2 \rangle_V^{(\pi)} + 3\,(1-\gamma)\,\frac{m_\rho^2}{(4\pi F)^2} \qquad \tilde{c}_2^{(\pi)} = 0 \tag{6}$$

$$\begin{aligned} c_3^{(\pi)} = & -\frac{m_\rho^2}{(4\pi F)^2}\Bigg[\frac{31}{24} - \gamma + 4\,\bar{L}_9 + \left(\frac{23}{36} - 2\,\bar{L}_9 + \frac{1}{12}\ell_K\right)\ell_\pi + \frac{5}{12}\ell_\pi^2 + \frac{5}{18}\ell_K + \frac{1}{8}\ell_K^2 \\ & + \left(\frac{5}{3} - \frac{2}{3}\gamma\right)\log\frac{m_\rho^2}{m_\pi^2} + \left(2 + 2\,\kappa^{(\pi)} - \frac{7}{3}\gamma\right)\log\frac{m_\rho^2}{\mu^2} + K^{(\pi)}(0)\Bigg] + c_3^{CT}(\mu) \end{aligned} \tag{7}$$

$$\begin{aligned} c_4^{(\pi)}(m_\ell) = & -\frac{m_\rho^2}{(4\pi F)^2}\Bigg\{\frac{z_\ell}{3(1-z_\ell)^2}\left[\Big(4(1-z_\ell) + (9-5z_\ell)\log z_\ell\Big) + 2\,\gamma\,\Big(1 - z_\ell + z_\ell \log z_\ell\Big)\right] \\ & + \left(\kappa^{(\pi)} + \frac{1}{3}\right)\frac{z_\ell}{2(1-z_\ell)}\,\log z_\ell + K^{(\pi)}(z_\ell) - K^{(\pi)}(0)\Bigg\} \end{aligned} \tag{8}$$

where $\kappa^{(\pi)}$ is related to the $O(p^4)$ pion charge radius by:

$$\kappa^{(\pi)} \equiv 4\,\bar{L}_9 - \frac{1}{6}\ell_K - \frac{1}{3}\ell_\pi - \frac{1}{2} = \frac{(4\pi F)^2}{3}\,\langle r^2\rangle_V^{(\pi)} \ . \tag{9}$$

The function $K^{(\pi)}(z_\ell)$, whose expression will be given in Ref. [10], does not contain any large logarithms and gives a small fractional contribution to $c_{3,4}^{(\pi)}$.

As anticipated, $c_2^{(\pi)}$ is a parameter-free prediction of ChPT. Moreover, we find $\tilde{c}_2^{(\pi)} = 0$, as expected due to a cancellation of real- and virtual-photon effects [15]. Finally, $c_3^{(\pi)}$ encodes calculable chiral corrections (as does $c_4(m_\ell)$) and a local counterterm $c_3^{CT}(\mu)$, for which our matching procedure [10] gives ($z_A \equiv m_{a_1}/m_\rho$):

$$\begin{aligned} c_3^{CT}(\mu) = & -\frac{19\,m_\rho^2}{9(4\pi F)^2} + \left(\frac{4\,m_\rho^2}{3(4\pi F)^2} + \frac{7+11z_A^2}{6z_A^2}\right)\log\frac{m_\rho^2}{\mu^2} \\ & + \frac{37 - 31z_A^2 + 17z_A^4 - 11z_A^6}{36z_A^2(1-z_A^2)^2} \\ & - \frac{7 - 5z_A^2 - z_A^4 + z_A^6}{3z_A^2(-1+z_A^2)^3}\,\log z_A \ . \end{aligned} \tag{10}$$

Numerically, using $z_A = \sqrt{2}$, we find $c_3^{CT}(m_\rho) = -1.61$, implying that the counterterm induces a sub-leading correction to c_3 (see Table I). The scale dependence of $c_3^{CT}(\mu)$ partially cancels the scale dependence of the chiral loops (our procedure captures all the "single-log" scale dependence). Taking a very conservative attitude we assign to c_3 an uncertainty equal to 100% of the local contribution ($|\Delta c_3| \sim 1.6$) plus the effect of residual renormalization scale dependence, obtained by varying the scale μ in the range 0.5 → 1 GeV ($|\Delta c_3| \sim 0.7$), leading to $\Delta c_3^{(\pi,K)} = \pm 2.3$. Full numerical values of $c_{2,3,4}^{(\pi)}$ are reported in Table I, with uncertainties due to matching procedure and input parameters (L_9 and γ [16]).

As a check on our calculation, we have verified that if we neglect c_3^{CT} and pure two-loop effects, and if we use $L_9 = F^2/(2m_\rho^2)$ (vector meson dominance), our results for $c_{2,3,4}^{(\pi)}$ are fully consistent with previous analyses of the leading structure dependent corrections based on current algebra [6, 17]. Moreover, our numerical value of $\Delta^{(\pi)}_{e^2p^4}$ reported in Table II is very close to the corresponding result in Ref. [6], $\Delta^{(\pi)}_{e^2p^4} = (0.054 \pm 0.044) \times 10^{-2}$.

For completeness we report here the contribution to $\Delta^{(\pi)}_{e^2p^6}$ induced by structure dependent radiation:

$$\begin{aligned} \Delta^{(\pi)}_{e^2p^6} = & \frac{\alpha}{2\pi}\,\frac{m_\pi^4}{(4\pi F)^4}\,(1+\gamma^2)\,\left[\frac{1}{30\,z_e} - \frac{11}{60} + \frac{z_e}{20(1-z_e)^2}\right. \\ & \left.\times\ \left(12 - 3z_e - 10z_e^2 + z_e^3 + 20\,z_e\log z_e\right)\right] . \end{aligned} \tag{11}$$

Results for $R^{(K)}_{e/\mu}$ - In this case we have:

$$c_2^{(K)} = \frac{2}{3}\,m_\rho^2\,\langle r^2\rangle_V^{(K)} + \frac{4}{3}\left(1 - \frac{7}{4}\gamma\right)\frac{m_\rho^2}{(4\pi F)^2} \tag{12}$$

$$\tilde{c}_2^{(K)} = \frac{1}{3}\,(1-\gamma)\,\frac{m_\rho^2}{(4\pi F)^2} \tag{13}$$

where $\langle r^2\rangle_V^{(K)}$ is the $O(p^4)$ kaon charge radius. $c_3^{(K)}$ is obtained from $c_3^{(\pi)}$ by replacing $31/24 - \gamma \to -7/72 -$

	$(P=\pi)$	$(P=K)$
$\tilde{c}_2^{(P)}$	0	$(7.84 \pm 0.07_\gamma) \times 10^{-2}$
$c_2^{(P)}$	$5.2 \pm 0.4_{L_9} \pm 0.01_\gamma$	$4.3 \pm 0.4_{L_9} \pm 0.01_\gamma$
$c_3^{(P)}$	$-10.5 \pm 2.3_{\rm m} \pm 0.53_{L_9}$	$-4.73 \pm 2.3_{\rm m} \pm 0.28_{L_9}$
$c_4^{(P)}(m_\mu)$	$1.69 \pm 0.07_{L_9}$	$0.22 \pm 0.01_{L_9}$

TABLE I: Numerical values of the coefficients $c_n^{(P)}$ of Eq. 3 $(P=\pi,K)$. The uncertainties correspond to the input values $L_9^r(\mu=m_\rho) = (6.9 \pm 0.7) \times 10^{-3}$, $\gamma = 0.465 \pm 0.005$ [16], and to the matching procedure (m), affecting only $c_3^{(P)}$.

	$(P=\pi)$	$(P=K)$
$\Delta_{e^2p^2}^{(P)}$ (%)	-3.929	-3.786
$\Delta_{e^2p^4}^{(P)}$ (%)	0.053 ± 0.011	0.135 ± 0.011
$\Delta_{e^2p^6}^{(P)}$ (%)	0.073	
Δ_{LL} (%)	0.055	0.055

TABLE II: Numerical summary of various electroweak corrections to $R_{e/\mu}^{(\pi,K)}$.

$13/9\,\gamma$, by dropping the term proportional to $\log m_\rho^2/m_\pi^2$, and by inter-changing everywhere else the label π with K (masses, $\ell_\pi \to \ell_K$, etc.). $c_4^{(K)}$ is obtained from $c_4^{(\pi)}$ by keeping only the second line of Eq. 8 and inter-changing the labels π and K. The numerical values of $c_{2,3,4}^{(K)}$ and $\tilde{c}_2^{(K)}$ are reported in Table I.

Resumming leading logarithms - At the level of uncertainty considered, one needs to include higher order long distance corrections to the leading contribution $\Delta_{e^2p^2} \sim -3\alpha/\pi \log m_\mu/m_e \sim -3.7\%$. The leading logarithms can be summed via the renormalization group and their effect amounts to multiplying $R_{e/\mu}^{(P)}$ by [6]

$$1 + \Delta_{LL} = \frac{\left(1 - \frac{2}{3}\frac{\alpha}{\pi}\log\frac{m_\mu}{m_e}\right)^{9/2}}{1 - \frac{3\alpha}{\pi}\log\frac{m_\mu}{m_e}} = 1.00055\ . \quad (14)$$

Conclusions - In Table II we summarize the various corrections to $R_{e/\mu}^{(\pi,K)}$, which lead to our final results:

$$R_{e/\mu}^{(\pi)} = (1.2352 \pm 0.0001) \times 10^{-4} \quad (15)$$

$$R_{e/\mu}^{(K)} = (2.477 \pm 0.001) \times 10^{-5}\ . \quad (16)$$

In the case of $R_{e/\mu}^{(K)}$ we have inflated the nominal uncertainty arising from matching by a factor of four, to account for higher order chiral corrections of expected size $\Delta_{e^2p^4} \times m_K^2/(4\pi F)^2$. Our results have to be compared with the ones of Refs. [6] and [7] reported in the introduction. While $R_{e/\mu}^{(\pi)}$ is in good agreement with both previous results, there is a discrepancy in $R_{e/\mu}^{(K)}$ that goes well outside the estimated theoretical uncertainties. We have traced back this difference to the following problems in Ref. [7]: (i) the leading log correction Δ_{LL} is included with the wrong sign (this accounts for half of the discrepancy); (ii) the NLO virtual correction $\Delta_{e^2p^4}^{(K)} = 0.058\%$ is not reliable because the hadronic form factors modeled in Ref. [7] do not satisfy the QCD short-distance behavior.

In conclusion, by performing the first ever ChPT calculation to $O(e^2p^4)$, we have improved the reliability of both the central value and the uncertainty of the ratios $R_{e/\mu}^{(\pi,K)}$. Our final result for $R_{e/\mu}^{(\pi)}$ is consistent with the previous literature, while we find a discrepancy in $R_{e/\mu}^{(K)}$, which we have traced back to inconsistencies in the analysis of Ref. [7]. Our results provide a clean basis to detect or constrain non-standard physics in these channels by comparison with upcoming measurements.

Acknowledgments – We wish to thank M. Ramsey-Musolf for collaboration at an early stage of this work, D. Pocanic and M. Bychkov for correspondence on the experimental input on γ, and W. Marciano and A. Sirlin for cross-checks on parts of our calculation. This work has been supported in part by the EU MRTN-CT-2006-035482 (FLAVIAnet), by MEC (Spain) under grant FPA2004-00996 and by Generalitat Valenciana under grant GVACOMP2007-156

[1] D. A. Bryman, Comm. Nucl. Part. Phys. **21**, 101 (1993).
[2] A. Masiero et al., Phys. Rev. D **74**, 011701 (2006); M. J. Ramsey-Musolf et al., arXiv:0705.0028 [hep-ph].
[3] PEN, PSI exp. R–05-01, 2006; PIENU, TRIUMF exp. 1072, D. Bryman, T. Numao, spokespersons (2006).
[4] NA48/3 at CERN; KLOE at DAFNE, INFN-Frascati.
[5] D.I. Britton et al, Phys. Rev. Lett. **68**, 3000(1992), Phys. Rev.Ḋ **49**, 28(1994); G. Czapek et al., Phys. Rev. Lett. **70**, 17 (1993).
[6] W. J. Marciano and A. Sirlin, Phys. Rev. Lett. **71**, 3629 (1993).
[7] M. Finkemeier, Phys. Lett. B **387**, 391 (1996).
[8] M. Knecht et al., Eur. Phys. J. C **12**, 469 (2000).
[9] V. Cirigliano et al., Eur. Phys. J. C **23**, 121 (2002); Eur. Phys. J. C **27**, 255 (2003); Eur. Phys. J. C **35**, 53 (2004).
[10] V. Cirigliano and I. Rosell, in preparation.
[11] T. Kinoshita, Phys. Rev. Lett. **2**, 477 (1959).
[12] J. Bijnens et al., Nucl. Phys. B **396**, 81 (1993).
[13] J. Gasser and H. Leutwyler, Nucl. Phys. B **250**, 465 (1985).
[14] B. Moussallam, Nucl. Phys. B **504**, 381 (1997); M. Knecht and A. Nyffeler, Eur. Phys. J. C **21**, 659 (2001); V. Cirigliano et al., Phys. Lett. B **596**, 96 (2004).
[15] W. J. Marciano and A. Sirlin, Phys. Rev. Lett. **36**, 1425 (1976).
[16] M. Bychkov and D. Pocanic, private communication.
[17] M. V. Terentev, Yad. Fiz. **18**, 870 (1973).