Pion leptonic decays and supersymmetry

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We compute supersymmetric contributions to pion leptonic (π_{l2}) decays in the minimal supersymmetric standard model (MSSM). When R-parity is conserved, the largest contributions to the ratio $R_{e/\mu} \equiv \Gamma[\pi^+ \to e^+ \nu_e(\gamma)]/\Gamma[\pi^+ \to \mu^+ \nu_\mu(\gamma)]$ arise from one-loop $(V - A) \otimes (V - A)$ corrections. These contributions can be potentially as large as the sensitivities of upcoming experiments; if measured, they would imply significant bounds on the chargino and slepton sectors complementary to current collider limits. We also analyze R-parity-violating interactions, which may produce a detectable deviation in $R_{e/\mu}$ while remaining consistent with all other precision observables.

I. INTRODUCTION

Low-energy precision tests provide important probes of new physics that are complementary to collider experiments [1–3]. In particular, effects of weak-scale supersymmetry (SUSY)—one of the most popular extensions of the standard model (SM)—can be searched for in a wide variety of low-energy tests: muon $(g - 2)$ [4], β- and μ-decay [5,6], parity-violating electron scattering [7], electric dipole moment searches [8], and SM-forbidden transitions like $\mu \to e\gamma$ [9], etc. (for a recent review, see Ref. [10]). In this paper, we compute the SUSY contributions to pion leptonic (π_{l2}) decays and analyze the conditions under which they can be large enough to produce observable effects in the next generation of experiments.

In particular, we consider the ratio $R_{e/\mu}$, defined by

$$R_{e/\mu} \equiv \frac{\Gamma(\pi^+ \to e^+ \nu_e + e^+ \nu_e \gamma)}{\Gamma(\pi^+ \to \mu^+ \nu_\mu + \mu^+ \nu_\mu \gamma)}. \qquad (1)$$

The key advantage of $R_{e/\mu}$ is that a variety of QCD effects that bring large theoretical uncertainties—such as the pion decay constant F_π and lepton flavor-independent QCD radiative corrections—cancel from this ratio. Indeed, $R_{e/\mu}$ is one of a few electroweak observables that involve hadrons and yet are precisely calculable (see [11] for discussion and Refs. [12,13] for explicit computations). Moreover, measurements of this quantity provide unique probes of deviations from lepton universality of the charged current (CC) weak interaction in the SM that are induced by loop corrections and possible extensions of the SM. In the present case, we focus on contributions from SUSY that can lead to deviations from lepton universality.

Until recently, the two most precise theoretical calculations of $R_{e/\mu}$ in the SM were [12,13]

$$R_{e/\mu}^{SM} = \begin{cases} (1.2352 \pm 0.0005) \times 10^{-4}, \\ (1.2354 \pm 0.0002) \times 10^{-4}, \end{cases} \qquad (2)$$

where the theoretical uncertainty comes from pion structure effects. Recently, by utilizing chiral perturbation theory, $R_{e/\mu}$ has been calculated with even better precision [14]: $R_{e/\mu}^{SM} = (1.2352 \pm 0.0001) \times 10^{-4}$. Experimentally, the most precise measurements of $R_{e/\mu}$ have been obtained at TRIUMF [15] and PSI [16]. Taking the average of these results gives [17]

$$R_{e/\mu}^{\text{EXPT}} = (1.230 \pm 0.004) \times 10^{-4}, \qquad (3)$$

in agreement with the SM. Future experiments at these facilities will make more precise measurements of $R_{e/\mu}$, aiming for precision at the level of $<1 \times 10^{-3}$ (TRIUMF [18]) and 5×10^{-4} (PSI [19]). These projected uncertainties are close to the conservative estimate of theoretical uncertainties given in Ref. [12].

Deviations $\Delta R_{e/\mu}$ from the SM predictions in Eq. (2) would signal the presence of new, lepton flavor-dependent physics. In the minimal supersymmetric standard model (MSSM), a nonvanishing $\Delta R_{e/\mu}^{\text{SUSY}}$ may arise from either tree-level or one-loop corrections. In Sec. II, we consider contributions to $\Delta R_{e/\mu}^{\text{SUSY}}$ arising from R-parity conserving interactions (Fig. 1). Although tree-level charged Higgs exchange can contribute to the rate $\Gamma[\pi^+ \to \ell^+ \nu(\gamma)]$, this correction is flavor independent and cancels from $R_{e/\mu}$. One-loop corrections induce both scalar and vector semileptonic dimension six four-fermion operators. Such interactions contribute via pseudoscalar and axial vector pion decay amplitudes, respectively. We show that the pseudoscalar contributions are negligible unless the ratio of the up- and down-type Higgs vacuum expectation values (vevs) is huge ($v_u/v_d \equiv \tan\beta \gtrsim 10^3$). For smaller $\tan\beta$ the most important effects arise from one-loop contributions to the axial vector amplitude, which we analyze in detail by performing a numerical scan over MSSM parameter space. We find that experimental observation of SUSY loop-induced deviations at a significant level would require further reductions in both the experimental error and theoretical SM uncertainty. Such improvements could lead to stringent tests of "slepton universality" of the charged current sector of the MSSM, for which it is often
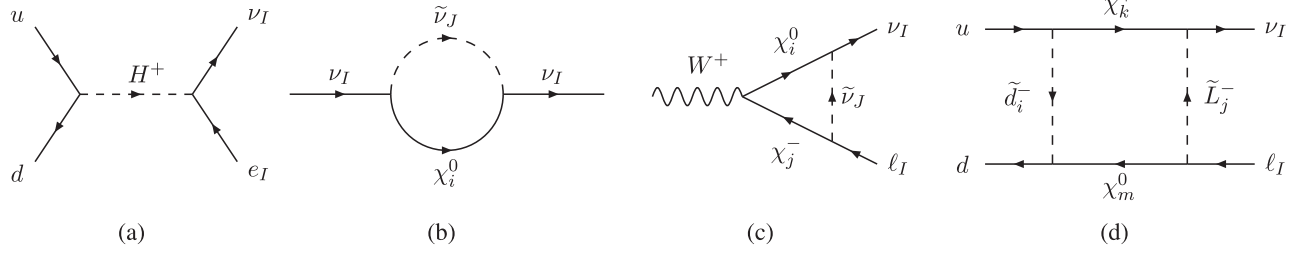
FIG. 1.   Representative contributions to $\Delta R^{\text{SUSY}}_{e/\mu}$: (a) tree-level charged Higgs boson exchange, (b) external leg diagrams, (c) vertex diagrams, (d) box diagrams. Graph (a) contributes to the pseudoscalar amplitude, graphs (b),(c) contribute to the axial vector amplitude, and graph (d) contributes to both amplitudes.

assumed that the left-handed first and second generation sleptons $\tilde{e}_L$ and $\tilde{\mu}_L$ are degenerate (see e.g. [20]) and thus $\Delta R^{\text{SUSY}}_{e/\mu} \simeq 0$.

In Sec. III, we consider corrections to $R_{e/\mu}$ from R-parity-violating (RPV) processes. These corrections enter at tree level, but are suppressed by couplings whose strength is constrained by other measurements. In order to analyze these constraints, we perform a fit to the current low-energy precision observables. We find that, at 95% C.L., the magnitude of RPV contributions to $\Delta R^{\text{SUSY}}_{e/\mu}$ could be several times larger than the combined theoretical and anticipated experimental errors for the future $R_{e/\mu}$ experiments. We summarize the main results and provide conclusions in Sec. IV. Details regarding the calculation of one-loop corrections are given in the appendix.

II. R-PARITY CONSERVING INTERACTIONS

A. Pseudoscalar contributions

The tree-level amplitude for $\pi^+ \rightarrow \ell^+ \nu_\ell$ that arises from the $(V - A) \otimes (V - A)$ four-fermion operator is

$$i\mathcal{M}^{(0)}_{AV} = -i2\sqrt{2}G_\mu V_{ud}\langle 0|\bar{d}\gamma^\lambda P_L u|\pi^+\rangle\bar{u}_\nu\gamma_\lambda P_L v_\ell$$
$$= 2V_{ud}F_\pi G_\mu m_\ell \bar{u}_\nu P_R v_\ell, \qquad (4)$$

where $P_{L,R}$ are the left- and right-handed projection operators,

$$F_\pi = 92.4 \pm 0.07 \pm 0.25 \text{ MeV} \qquad (5)$$

is the pion decay constant, G_μ is the Fermi constant extracted from the muon lifetime, and V_{ud} is the (1, 1) component of the Cabibbo-Kobayashi-Maskawa (CKM) matrix. The first error in Eq. (5) is experimental while the second arises from uncertainties associated with QCD effects in the one-loop SM electroweak radiative corrections to the $\pi_{\mu 2}$ decay rate. The superscript "(0)" and subscript "AV" in Eq. (4) denote a tree-level, axial vector contribution. At one-loop order, one must subtract the radiative corrections to the muon-decay amplitude—since G_μ is obtained from the muon lifetime—while adding the corrections to the semileptonic CC amplitude. The corrections to the muon-decay amplitude as well as lepton-flavor-

independent contributions to the semileptonic radiative corrections cancel from $R_{e/\mu}$.

Now consider the contribution from an induced pseudoscalar four-fermion effective operator of the form

$$\Delta\mathcal{L}_{PS} = -\frac{G_{PS}V_{ud}}{\sqrt{2}}\bar{\nu}(1 + \gamma^5)\ell\bar{b}\gamma^5 u. \qquad (6)$$

Contributions to $R_{e/\mu}$ from operators of this form were considered in a model-independent operator framework in Ref. [21] and in the MSSM in Ref. [22]. In the MSSM, such an operator can arise at tree level [Fig. 1(a)] through charged Higgs exchange and at one-loop through box graphs [Fig. 1(d)]. These amplitudes determine the value of G_{PS}. The total amplitude is

$$i\mathcal{M}^{(0)}_{AV} + i\mathcal{M}_{PS} = V_{ud}F_\pi G_\mu m_\ell \bar{u}_\nu(1 + \gamma^5)$$
$$\times v_\ell\left[1 + \frac{G_{PS}}{G_\mu}\omega_\ell\right] \qquad (7)$$

where

$$\omega_\ell \equiv \frac{m_\pi^2}{m_\ell(m_u + m_d)} \simeq \begin{cases} 5 \times 10^3 & \ell = e \\ 20 & \ell = \mu \end{cases} \qquad (8)$$

is an enhancement factor reflecting the absence of helicity suppression in pseudoscalar contributions as compared to $(V - A) \otimes (V - A)$ contributions [23]. Pseudoscalar contributions will be relevant to the interpretation of $R_{e/\mu}$ if

$$\left|\frac{G_{PS}}{G_\mu}\right|\omega_\ell \gtrsim 0.0005, \qquad (9)$$

and if $G_{PS}\omega_\ell$ is lepton flavor dependent.

The tree-level pseudoscalar contribution [Fig. 1(a)] gives

$$G^{(0)}_{PS} = \frac{m_\ell \tan\beta(m_u \cot\beta - m_d \tan\beta)}{\sqrt{2}m^2_{H^+}v^2}, \qquad (10)$$

where m_{H^+} is the mass of the charged Higgs boson. Thus, we have

$$\frac{G^{(0)}_{PS}}{G_\mu}\omega_\ell = \frac{m_\pi^2 \tan\beta(m_u \cot\beta - m_d \tan\beta)}{(m_u + m_d)m^2_{H^+}}. \qquad (11)$$

It is indeed possible to satisfy (9) for

$$\tan\beta \gtrsim 20\left(\frac{m_{H^+}}{100 \text{ GeV}}\right). \tag{12}$$

Note that the combination $G_{PS}^{(0)}/G_\mu \times \omega_\ell$ entering Eq. (7) is independent of lepton flavor and will cancel from $R_{e/\mu}$. In principle, however, the extraction of F_π from $\pi_{\mu2}$ decay could be affected by tree-level charged Higgs exchange if the correction in Eq. (9) is $\gtrsim 0.003$ in magnitude, corresponding to a shift comparable to the theoretical SM uncertainty as estimated in Ref. [12]. In the case of charged Higgs exchange, one would require $\tan\beta \gtrsim 120(m_{H^+}/100 \text{ GeV})$ to generate such an effect.

One-loop contributions to G_{PS} are generated by box graphs [Fig. 1(d)]. The magnitude of these contributions is governed by the strength of chiral symmetry breaking in both the quark and lepton sectors. Letting ϵ generically denote either a Yukawa coupling y_f or a ratio m_f/M_{SUSY} (where $f = e, \mu, u,$ or d), we find that

$$\frac{G_{PS}^{(1)}}{G_\mu} \sim \frac{\alpha}{8\pi s_W^2}\left(\frac{m_W}{M_{SUSY}}\right)^2 \epsilon^2, \tag{13}$$

where the superscript "(1)" denotes one loop-induced pseudoscalar interaction. We have verified by explicit computation that the $\mathcal{O}(\epsilon)$ contributions vanish. The reason is that in each pair of incoming quarks or outgoing leptons the two fermions must have opposite chirality in order to contribute to $G_{PS}^{(1)}$. Since CC interactions in the MSSM are purely left-handed, the chirality must change at least twice in each graph, with each flip generating a factor of ϵ. For example, we show one pseudoscalar contribution in Fig. 2 that is proportional to $\epsilon^2 = y_\mu y_d$. Here, the chirality changes at the $u\tilde{d}\tilde{H}$ and $\nu\tilde{\mu}\tilde{H}$ vertices. Potentially, this particular contribution can be enhanced for large $\tan\beta$; however, to satisfy (9), we need

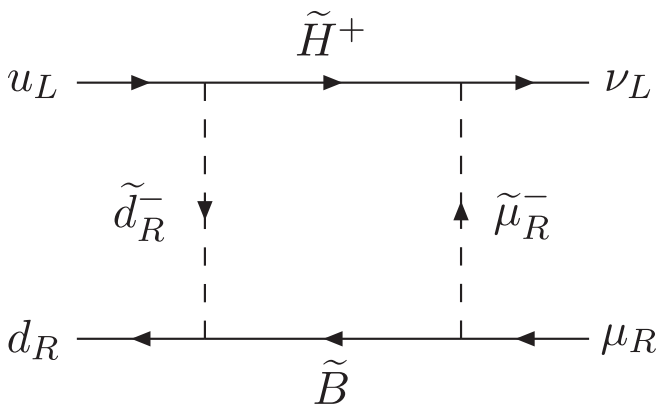$$\tan\beta \gtrsim 10^3\left(\frac{M_{SUSY}}{100 \text{ GeV}}\right)^3. \tag{14}$$

FIG. 2. This contribution to $G_{PS}^{(1)}$ is suppressed by $\epsilon^2 = y_\mu y_d$.

These extreme values of $\tan\beta$ can be problematic, leading y_b and y_τ to become nonperturbatively large. To avoid this scenario, we need roughly $\tan\beta \lesssim 65$ (see [20] and references therein).

Pseudoscalar contributions can also arise through mixing of left- and right-handed scalar superpartners. Since each left-right mixing insertion introduces a factor of ϵ, the leading contributions to $G_{PS}^{(1)}$ will still be $\mathcal{O}(\epsilon^2)$. However, if the triscalar SUSY-breaking parameters a_f are not suppressed by y_f as normally assumed, it is possible to have $\epsilon \sim \mathcal{O}(1)$, potentially leading to significant contributions. This possibility, although not experimentally excluded, is considered theoretically "unnatural" as it requires some fine-tuning to avoid spontaneous color and charge breaking (see Ref. [6] for discussion). Neglecting this possibility and extremely large values of $\tan\beta$, we conclude that loop-induced pseudoscalar contributions are much too small to be detected at upcoming experiments. These conclusions are consistent with an earlier, similar analysis in Ref. [22].

B. Axial vector contributions

One-loop radiative corrections also contribute to the axial vector amplitude. The total amplitude can be written as

$$i\mathcal{M}_{AV} = V_{ud}f_\pi G_\mu m_\ell \bar{u}_\nu(1 + \gamma^5)v_\ell[1 + \Delta\hat{r}_\pi - \Delta\hat{r}_\mu], \tag{15}$$

where $\Delta\hat{r}_\pi$ and $\Delta\hat{r}_\mu$ denote one-loop contributions to the semileptonic and μ-decay amplitudes, respectively, and where the hat indicates quantities renormalized in the modified dimensional reduction ($\overline{DR}$) scheme. Since $\Delta\hat{r}_\mu$ cancels from $R_{e/\mu}$, we concentrate on the SUSY contributions to $\Delta\hat{r}_\pi$ that do not cancel from $R_{e/\mu}$. It is helpful to distinguish various classes of contributions

$$\Delta\hat{r}_\pi^{\text{SUSY}} = \Delta_L^\ell + \Delta_V^\ell + \Delta_L^q + \Delta_V^q + \Delta_B + \Delta_{GB}, \tag{16}$$

where Δ_L^ℓ (Δ_L^q), Δ_V^ℓ (Δ_V^q), Δ_B, and Δ_{GB} denote leptonic (hadronic) external leg [Fig. 1(b)], leptonic (hadronic) vertex [Fig. 1(c)], box graph [Fig. 1(d)], and gauge boson propagator contributions, respectively. The corrections $\Delta_{L,V}^q$ and Δ_{GB} cancel from $R_{e/\mu}$, so we do not discuss them further (we henceforth omit the "ℓ" superscript). The explicit general formulas for $\Delta_{L,V,B}$, calculated in $\overline{DR}$, are given in the appendix. We have verified that Δ_L and Δ_V agree with Ref. [24] for the case of a pure $SU(2)_L$ chargino/neutralino sector.

At face value, it appears from Eqs. (A7)–(A9) that $\Delta R_{e/\mu}^{\text{SUSY}}$ carries a nontrivial dependence on MSSM parameters since the SUSY masses enter both explicitly in the loop functions and implicitly in the mixing matrices Z, defined in Eqs. (A1)–(A6). Nevertheless, we are able to identify a relatively simple dependence on the SUSY spectrum.

We first consider $\Delta R_{e/\mu}^{\text{SUSY}}$ in a limiting case obtained with three simplifying assumptions: (1) no flavor mixing among scalar superpartners; (2) no mixing between left- and right-handed scalar superpartners; and (3) degeneracy between $\tilde{\ell}_L$ and $\tilde{\nu}_\ell$ and no gaugino-Higgsino mixing. Our first assumption is well justified; experimental bounds on flavor violating processes constrain the contributions to $R_{e/\mu}$ from lepton flavor violation in the slepton soft-breaking sector to be less than the sensitivities at upcoming experiments by a factor of $10 - 20$ [22].

Our second assumption has minimal impact. In the absence of flavor mixing, the charged slepton mass matrix decomposes into three 2×2 blocks; thus, for flavor ℓ, the mass matrix in the $\{\tilde{\ell}_L, \tilde{\ell}_R\}$ basis is

$$\begin{pmatrix} M_L^2 + (s_W^2 - \frac{1}{2})m_Z^2 \cos2\beta & m_\ell(\frac{a_\ell}{y_\ell} - \mu\tan\beta) \\ m_\ell(\frac{a_\ell}{y_\ell} - \mu\tan\beta) & M_R^2 - s_W^2 m_Z^2 \cos2\beta \end{pmatrix},$$

where M_L^2 (M_R^2) is the SUSY-breaking mass parameter for left-handed (right-handed) sleptons, a_ℓ is the coefficient for the SUSY-breaking triscalar interaction, y_ℓ is the Yukawa coupling, and μ is the Higgsino mass parameter. Under particular models of SUSY-breaking mediation, it is usually assumed that $a_\ell/y_\ell \sim M_{\text{SUSY}}$, and thus left-right mixing is negligible for the first two generations due to the smallness of m_e and m_μ. Of course, a_ℓ could be significantly larger and induce significant left-right mixing [6]. For reasons discussed above, we neglect this possibility.

We have adopted the third assumption for purely illustrative purposes; we will relax it shortly. Clearly, fermions of the same weak isospin doublet are not degenerate; their masses obey

$$m_{\tilde{\ell}_L}^2 = m_{\tilde{\nu}_\ell}^2 - m_W^2 \cos2\beta + m_\ell^2 \qquad (17)$$

$$m_{\tilde{d}_L}^2 = m_{\tilde{u}_L}^2 - m_W^2 \cos2\beta + m_d^2 - m_u^2. \qquad (18)$$

In addition, gaugino mixing is certainly always present, as the gaugino mass matrices contain off-diagonal elements proportional to m_Z [see Eqs. (A2) and (A4)]. However, the third assumption becomes valid for $M_{\text{SUSY}} \gg m_Z$.

Under our three assumptions, the SUSY vertex and external leg corrections sum to a constant that is independent of the superpartner masses, leading to considerable simplifications. The bino [U(1)$_Y$ gaugino] vertex and external leg corrections exactly cancel. The wino [SU(2)$_L$ gaugino] vertex and leg corrections do not cancel; rather, $\Delta_V + \Delta_L = \alpha/4\pi s_W^2$, a constant that carries no dependence on the slepton, gaugino, or Higgsino mass parameters. The occurrence of this constant is merely an artifact of our use of the $\overline{\text{DR}}$ renormalization scheme. (In comparison, in modified minimal subtraction, we find $\Delta_V + \Delta_L = 0$ in this same limit.[1]) This dependence on the renormalization

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[1] Technically, since $\overline{MS}$ breaks SUSY, it is not the preferred renormalization scheme for the MSSM. However, this aspect is not important in the present calculation.

scheme cancels in $R_{e/\mu}$. [In addition, this scheme-dependent constant enters into the extraction of G_μ; hence, the individual decay widths $\Gamma(\pi \to \ell\nu_\ell)$ are also independent of the renormalization scheme.]

The reason for this simplification is that under our assumptions, we have effectively taken a limit that is equivalent to computing the one-loop corrections in the absence of electroweak symmetry breaking. In the limit of unbroken SU(2)$_L \times$ U(1)$_Y$, the one-loop SUSY vertex and external leg corrections sum to a universal constant which is renormalization scheme dependent, but renormalization scale independent [24]. [For unbroken SU(2)$_L$, the SM vertex and external leg corrections yield an additional logarithmic scale dependence; hence, the SU(2)$_L$ β-function receives contributions from both charge and wave function renormalization.] In addition, virtual Higgsino contributions are negligible, since their interactions are suppressed by small first and second generation Yukawa couplings. Setting all external momenta to zero and working in the limit of unbroken SU(2)$_L$ symmetry, we find that the Higgsino contributions to $\Delta_L + \Delta_V$ are $y_\ell^2/32\pi^2$.

In this illustrative limit, the only nonzero contributions to $\Delta R_{e/\mu}^{\text{SUSY}}$ come from two classes of box graphs [Fig. 1(d)]—one involving purely wino-like interactions and the other with both a virtual wino and bino. The sum of these graphs is

$$\Delta_B^{(\ell)} = \frac{\alpha}{12\pi s_W^2}\left(\frac{m_W^2}{M_2^2}\right)[F_1(x_L, x_Q) + t_W^2 F_2(x_B, x_L, x_Q)] \qquad (19)$$

where we have defined

$$F_1(x_L, x_Q) \equiv \frac{3}{2}\left[\frac{x_L(x_L - 2)\ln x_L}{(x_L - x_Q)(1 - x_L)^2} + \frac{x_Q(x_Q - 2)\ln x_Q}{(x_Q - x_L)(1 - x_Q)^2} - \frac{1}{(1 - x_L)(1 - x_Q)}\right] \qquad (20)$$

and

$$F_2(x_B, x_L, x_Q) \equiv \frac{1}{2}\left[\frac{x_B(x_B + 2\sqrt{x_B})\ln x_B}{(1 - x_B)(x_B - x_L)(x_B - x_Q)} + \frac{x_L(x_L + 2\sqrt{x_B})\ln x_L}{(1 - x_L)(x_L - x_B)(x_L - x_Q)} + \frac{x_Q(x_Q + 2\sqrt{x_B})\ln x_Q}{(1 - x_Q)(x_Q - x_L)(x_Q - x_B)}\right], \qquad (21)$$

where $x_B \equiv M_1^2/M_2^2$, $x_L \equiv m_{\tilde{\ell}}^2/M_2^2$, and $x_Q \equiv m_{\tilde{Q}}^2/M_2^2$, with masses M_1, M_2, $m_{\tilde{\ell}}$, and $m_{\tilde{Q}}$ of the bino, wino, left-handed ℓ-flavored slepton, and left-handed 1st generation squark, respectively. Numerically, we find that always $F_1 \gg F_2$; the reason is that the sum of bino-wino graphs
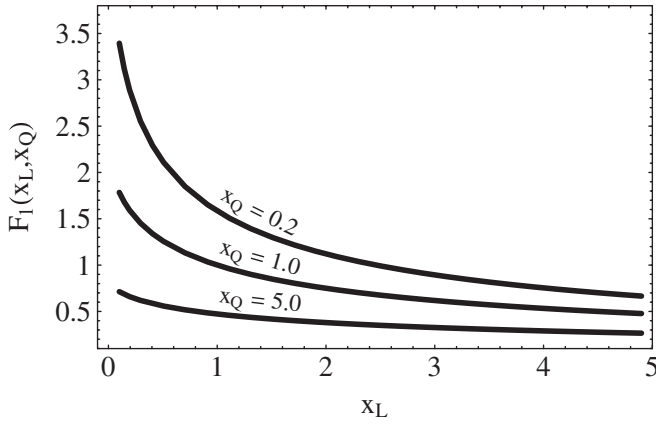
FIG. 3. The box graph loop function $F_1(x_L, x_Q)$ as a function of $x_L \equiv m_{\tilde{L}}^2 / M_2^2$ for several values of $x_Q \equiv m_{\tilde{Q}}^2 / M_2^2$. For $x_L \sim x_Q \sim 1$ (i.e. SUSY masses degenerate), $F_1(x_L, x_Q) \sim 1$. For $x_L \gg 1$ or $x_Q \gg 1$ (i.e. very massive sleptons or squarks), $F_1(x_L, x_Q) \to 0$.

tend to cancel, while the sum of pure wino graphs all add coherently. Hence, bino exchange (through which the term proportional to F_2 arises) does not significantly contribute to $\Delta R_{e/\mu}^{\rm SUSY}$.

In Fig. 3, we show $F_1(x_L, x_Q)$ as a function of x_L for fixed x_Q. Since F_1 is symmetric under $x_L \leftrightarrow x_Q$, Fig. 3 also shows F_1 as a function of x_Q, and hence how Δ_B depends on $m_{\tilde{u}_L}$. For $x_L, x_Q \sim 1$, we have $F_1 \sim \mathcal{O}(1)$, while if either $x_L \gg 1$ or $x_Q \gg 1$, then $F_1 \to 0$, which corresponds to the decoupling of heavy sleptons or squarks. There is no enhancement of Δ_B for $x_L \ll 1$ or $x_Q \ll 1$ (i.e. if M_2 is very heavy) due to the overall $1/M_2^2$ suppression in (19).

The total box graph contribution is

$$\frac{\Delta R_{e/\mu}^{\rm SUSY}}{R_{e/\mu}^{\rm SM}} = 2\,{\rm Re}[\Delta_B^{(e)} - \Delta_B^{(\mu)}]$$

$$\simeq \frac{\alpha}{6\pi s_W^2}\left(\frac{m_W}{M_2}\right)^2\left[F_1\left(\frac{m_{\tilde{e}}^2}{M_2^2}, \frac{m_{\tilde{Q}}^2}{M_2^2}\right) - F_1\left(\frac{m_{\tilde{\mu}}^2}{M_2^2}, \frac{m_{\tilde{Q}}^2}{M_2^2}\right)\right].$$
$$(22)$$

Clearly $\Delta R_{e/\mu}^{\rm SUSY}$ vanishes if both sleptons are degenerate and is largest when they are far from degeneracy, such that $m_{\tilde{e}_L} \gg m_{\tilde{\mu}_L}$ or $m_{\tilde{e}_L} \ll m_{\tilde{\mu}_L}$. In the latter case, we have

$$\left|\frac{\Delta R_{e/\mu}^{\rm SUSY}}{R_{e/\mu}^{\rm SM}}\right| \lesssim 0.001 \times \left(\frac{100\ {\rm GeV}}{M_{SUSY}}\right)^2 \qquad (23)$$

for e.g. $M_{\rm SUSY} \equiv M_2 \sim m_{\tilde{u}_L} \sim m_{\tilde{e}_L} \ll m_{\tilde{\mu}_L}$.

We now relax our third assumption to allow for gaugino-Higgsino mixing and nondegeneracy of $\tilde{\ell}$ and $\tilde{\nu}_\ell$. Both of these effects tend to spoil the universality of $\Delta_V + \Delta_L$, giving

$$\Delta_V + \Delta_L - \frac{\alpha}{4\pi s_W^2} \equiv \frac{\alpha}{8\pi s_W^2} f \simeq 0.001 f. \qquad (24)$$

The factor f measures the departure of $\Delta_V + \Delta_L$ from universality. If the SUSY spectrum is such that our third assumption is valid, we expect $f \to 0$. For realistic values of the SUSY parameters, two effects lead to a nonvanishing f: (a) splitting between the masses of the charged and neutral left-handed sleptons that results from breaking of $SU(2)_L$, and (b) gaugino-Higgsino mixing. The former effect is typically negligible. To see why, we recall from Eq. (18) that

$$m_{\tilde{\ell}} = m_{\tilde{\nu}_\ell}\left[1 + \mathcal{O}\left(\frac{m_W^2}{m_{\tilde{\ell}}^2}\right)\right], \qquad (25)$$

where we have neglected the small nondegeneracy proportional to the square of the lepton Yukawa coupling. We find that the leading contribution to f from this nondegeneracy is at least $\mathcal{O}(m_W^4/m_{\tilde{\ell}}^4)$, which is $\lesssim 0.1$ for $m_{\tilde{\ell}} \gtrsim 2M_W$.

Significant gaugino mixing can induce $f \sim \mathcal{O}(1)$. The crucial point is that the size of f from gaugino mixing is governed by the size of M_2. If $M_2 \gg m_Z$, then the wino decouples from the bino and Higgsino, and contributions to $\Delta_V + \Delta_L$ approach the case of unbroken $SU(2)_L$. On the other hand, if $M_2 \sim m_Z$, then $\Delta_V + \Delta_L$ can differ substantially from $\alpha/4\pi s_W^2$.

In the limit that $m_{\tilde{\ell}_L} \gg M_2$ ($\ell = e, \mu$), we also have a decoupling scenario where $\Delta_B = 0$, $\Delta_V + \Delta_L = \frac{\alpha}{4\pi s_W^2}$, and thus $f = 0$. Hence, a significant contribution to $\Delta R_{e/\mu}$ requires at least one light slepton. However, regardless of the magnitude of f, if $m_{\tilde{e}_L} = m_{\tilde{\mu}_L}$, then these corrections will cancel from $R_{e/\mu}$.

It is instructive to consider the dependence of individual contributions Δ_B and $\Delta_V + \Delta_L$ to $\Delta R_{e/\mu}^{\rm SUSY}$, as shown in
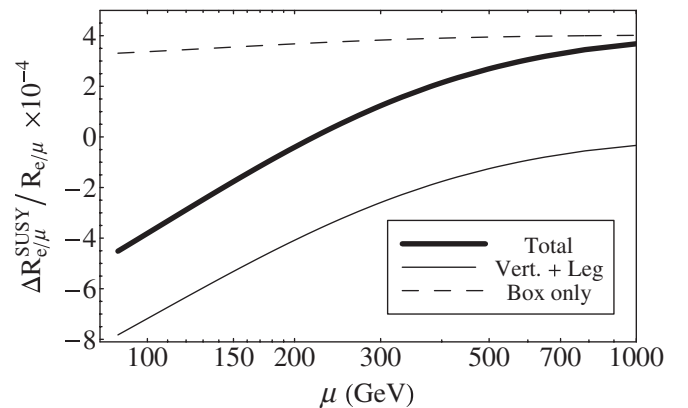

FIG. 4. $\Delta R_{e/\mu}^{\rm SUSY}$ versus μ, with fixed parameters $M_1 = 100$ GeV, $M_2 = 150$ GeV, $m_{\tilde{e}_L} = 100$ GeV, $m_{\tilde{\mu}_L} = 500$ GeV, $m_{\tilde{u}_L} = 200$ GeV. The thin solid line denotes contributions from $(\Delta_V + \Delta_L)$ only; the dashed line denotes contributions from Δ_B only; the thick solid line shows the sum of both contributions to $\Delta R_{e/\mu}^{\rm SUSY}$.
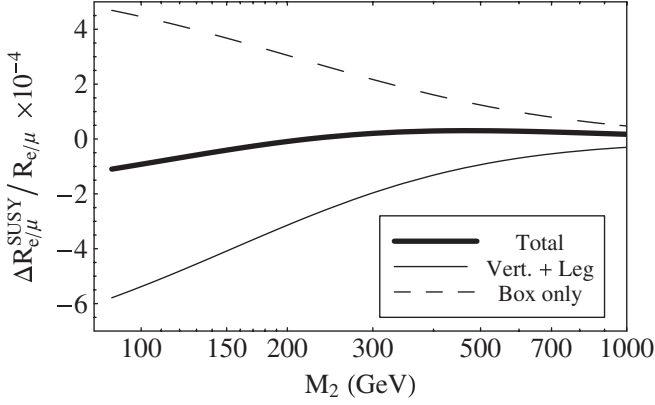
FIG. 5. $\Delta R_{e/\mu}^{\text{SUSY}}/R_{e/\mu}^{\text{SM}}$ as a function of M_2, with $\mu = 200$ GeV and all other parameters fixed as in Fig. 4. Each line shows the contribution indicated as in the caption of Fig. 4.

Figs. 4 and 5. In Fig. 4, we plot the various contributions as a function of the supersymmetric mass parameter μ, with $M_1 = 100$ GeV, $M_2 = 150$ GeV, $m_{\tilde{e}_L} = 100$ GeV, $m_{\tilde{\mu}_L} = 500$ GeV, $m_{\tilde{u}_L} = 200$ GeV. We see that the $\Delta_V + \Delta_L$ contributions (thin solid line) vanish for large μ, since in this regime gaugino-Higgsino mixing is suppressed and there is no $\Delta_V + \Delta_L$ contribution to $\Delta R_{e/\mu}^{\text{SUSY}}$. However, the Δ_B contribution (dashed line) is nearly μ independent, since box graphs with Higgsino exchange (which depend on μ) are suppressed in comparison to those with only gaugino exchange. In Fig. 5, we plot these contributions as a function of M_2, with $\mu = 200$ GeV and all other parameters fixed as above. We see that both $\Delta_V + \Delta_L$ and Δ_B contributions vanish for large M_2.

One general feature observed from these plots is that $\Delta_V + \Delta_L$ and Δ_B contributions tend to cancel one another; therefore, the largest total contribution to $\Delta R_{e/\mu}^{\text{SUSY}}$ occurs when either $\Delta_V + \Delta_L$ or Δ_B is suppressed in comparison to the other. This can occur in the following ways: (1) if $\mu \gg m_Z$, then Δ_B may be large, while $\Delta_V + \Delta_L$ is suppressed, and (2) if $m_{\tilde{u}_L}, m_{\tilde{d}_L} \gg m_Z$, then $\Delta_V + \Delta_L$ may be large, while Δ_B is suppressed. In Fig. 5, we have chosen parameters for which there is a large cancellation between $\Delta_V + \Delta_L$ and Δ_B. However, by taking the limits $\mu \to \infty$ or $m_{\tilde{u}_L}, m_{\tilde{d}_L} \to \infty$, $\Delta R_{e/\mu}^{\text{SUSY}}$ would coincide with the Δ_B or $\Delta_V + \Delta_L$ contributions, respectively.

Because the $\Delta_V + \Delta_L$ and Δ_B contributions tend to cancel, it is impossible to determine whether $\tilde{e}_L$ or $\tilde{\mu}_L$ is heavier from $R_{e/\mu}$ measurements alone. For example, a positive deviation in $R_{e/\mu}$ can result from two scenarios: (1) $\Delta R_{e/\mu}^{\text{SUSY}}$ is dominated by box graph contributions with $m_{\tilde{e}_L} < m_{\tilde{\mu}_L}$, or (2) $\Delta R_{e/\mu}^{\text{SUSY}}$ is dominated by $\Delta_V + \Delta_L$ contributions with $m_{\tilde{e}_L} > m_{\tilde{\mu}_L}$.

Guided by the preceding analysis, we expect for $\Delta R_{e/\mu}^{\text{SUSY}}$:

(i) The maximum contribution is $|\Delta R_{e/\mu}^{\text{SUSY}}/R_{e/\mu}| \sim$

0.001.

(ii) Both the vertex + leg and box contributions are largest if $M_2 \sim \mathcal{O}(m_Z)$ and vanish if $M_2 \gg m_Z$. If $M_2 \sim \mathcal{O}(m_Z)$, then at least one chargino must be light.

(iii) The contributions to $\Delta R_{e/\mu}^{\text{SUSY}}$ vanish if $m_{\tilde{e}_L} = m_{\tilde{\mu}_L}$ and are largest if either $m_{\tilde{\mu}_L} \ll m_{\tilde{e}_L}$ or $m_{\tilde{\mu}_L} \gg m_{\tilde{e}_L}$.

(iv) The contributions to $\Delta R_{e/\mu}^{\text{SUSY}}$ are largest if $\tilde{e}_L$ or $\tilde{\mu}_L$ is $\mathcal{O}(m_Z)$.

(v) If $\mu \gg m_Z$, then the lack of gaugino-Higgsino mixing suppresses the $\Delta_V + \Delta_L$ contributions to $\Delta R_{e/\mu}^{\text{SUSY}}$.

(vi) If $m_{\tilde{u}_L}, m_{\tilde{d}_L} \gg m_Z$, then the Δ_B contributions to $\Delta R_{e/\mu}^{\text{SUSY}}$ are suppressed due to squark decoupling.

(vii) If $\tilde{u}_L, \tilde{d}_L$, and μ are all $\mathcal{O}(m_Z)$, then there may be cancellations between the $\Delta_V + \Delta_L$ and Δ_B contributions. $\Delta R_{e/\mu}^{\text{SUSY}}$ is largest if it is dominated by *either* $\Delta_V + \Delta_L$ *or* Δ_B contributions.

We now study $\Delta R_{e/\mu}^{\text{SUSY}}$ quantitatively by making a numerical scan over MSSM parameter space, using the following ranges:

$$m_Z/2 < \{M_1, |M_2|, |\mu|, m_{\tilde{u}_L}\} < 1 \text{ TeV}$$
$$m_Z/2 < \{m_{\tilde{\nu}_e}, m_{\tilde{\nu}_\mu}\} < 5 \text{ TeV}$$
$$1 < \tan\beta < 50 \tag{26}$$
$$\text{sign}(\mu), \qquad \text{sign}(M_2) = \pm 1,$$

where $m_{\tilde{e}_L}$, $m_{\tilde{\mu}_L}$, and $m_{\tilde{d}_L}$ are determined from Eqs. (17) and (18).

Direct collider searches impose some constraints on the parameter space. Although the detailed nature of these constraints depends on the adoption of various assumptions and on interdependencies on the nature of the MSSM and its spectrum [17], we implement them in a coarse way in order to identify the general trends in corrections to $R_{e/\mu}$. First, we include only parameter points in which there are no SUSY masses lighter than $m_Z/2$. (However, the current bound on the mass of lightest neutralino is even weaker than this.) Second, parameter points which have no charged SUSY particles lighter than 103 GeV are said to satisfy the "LEP II bound." (This bound may also be weaker, in particular, regions of parameter space.)

Additional constraints arise from precision electroweak data. We consider only MSSM parameter points whose contributions to the oblique parameters S, T, and U [25] agree with electroweak precision observables (EWPO). A recent fit to both high- and low-energy EWPO using the value of $m_t = 170.9 \pm 1.8$ GeV [26] has been reported in Ref. [27], yielding

$$T = -0.111 \pm 0.109 \qquad S = -0.126 \pm 0.096$$
$$U = 0.164 \pm 0.115 \tag{27}$$

where the errors quoted are 1 standard deviation and where the value of the standard model Higgs boson mass has been set to the LEP lower bound $m_h = 114.4$ GeV. Using the correlation matrix given in Ref. [27] and the computation of superpartner contributions to the oblique parameters reported in Ref. [7], we determine the points in the MSSM parameter space that are consistent with EWPO at 95% confidence. Because we have neglected the 3rd generation and right-handed scalar sectors in our analysis and parameter scan, we do not calculate the entire MSSM contributions to S, T, and U. Rather, we only include those from charginos, neutralinos, and the first two generation left-handed scalar superpartners. Although incomplete, this serves as a conservative lower bound; in general, the contributions to S, T, and U from the remaining scalar superpartners (that we neglect) only cause further deviations from the measured values of the oblique parameters. In addition, we have assumed that the lightest CP-even Higgs mass is the same as the SM Higgs mass reference point: $m_h = 114.4$ GeV, neglecting the corrections due to the small mass difference, and the typically small contributions from the remaining heavier Higgs bosons.

We do not impose other electroweak constraints in the present study, but note that they will generally lead to further restrictions. For example, the results of the E821 measurement of the muon anomalous magnetic moment [28] tend to favor a positive sign for the μ parameter and relatively large values of $\tan\beta$. Eliminating the points with $\text{sign}(\mu) = -1$ will exclude half the parameter space in our scan, but the general trends are unaffected.

We show the results of our numerical scan in Figs. 6–9. In Figs. 6–8, the dark regions contain all MSSM parameter
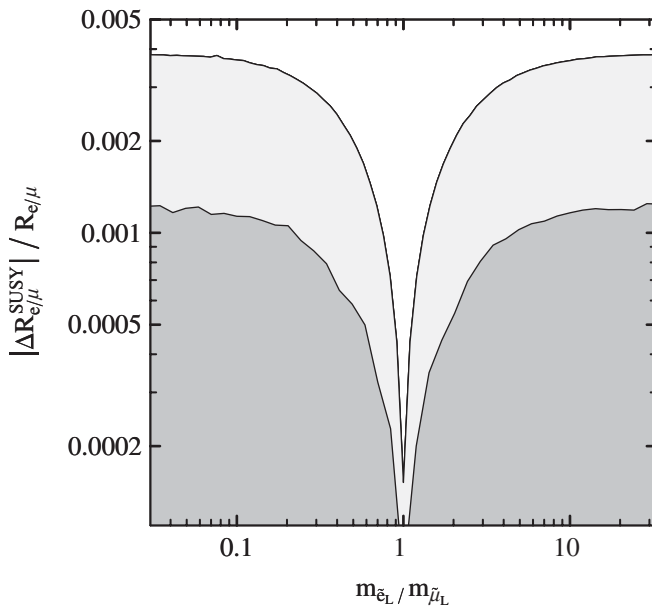

FIG. 7. $\Delta R_{e/\mu}^{\text{SUSY}}$ as a function of $\text{Min}[m_{\tilde{e}_L}, m_{\tilde{\mu}_L}]$, the mass of the lightest first or second generation charged slepton. The dark and light regions denote the regions of MSSM parameter space consistent and inconsistent, respectively, with the LEP II bound.

points within our scan consistent with the LEP II bound, while the light regions contain all MSSM points inconsistent with the LEP II bound, but with no superpartners lighter than $m_Z/2$. In effect, the dark (light) regions show how large $\Delta R_{e/\mu}^{\text{SUSY}}/R_{e/\mu}$ can be, assuming consistency (inconsistency) with the LEP II bound, as a function of a given parameter. In Fig. 6, we show $\Delta R_{e/\mu}^{\text{SUSY}}/R_{e/\mu}$ as a function of the ratio of slepton masses $m_{\tilde{e}_L}/m_{\tilde{\mu}_L}$. If both

FIG. 6. $\Delta R_{e/\mu}^{\text{SUSY}}$ as a function of the ratio $m_{\tilde{e}_L}/m_{\tilde{\mu}_L}$. The dark and light regions denote the regions of MSSM parameter space consistent and inconsistent, respectively, with the LEP II bound.
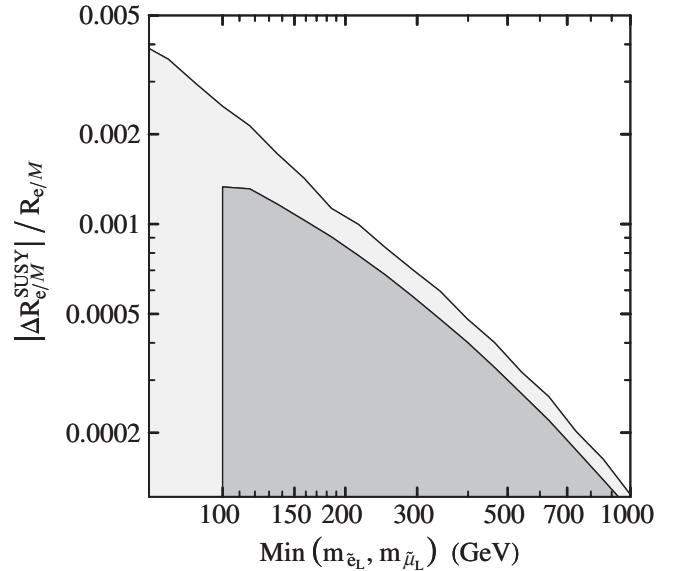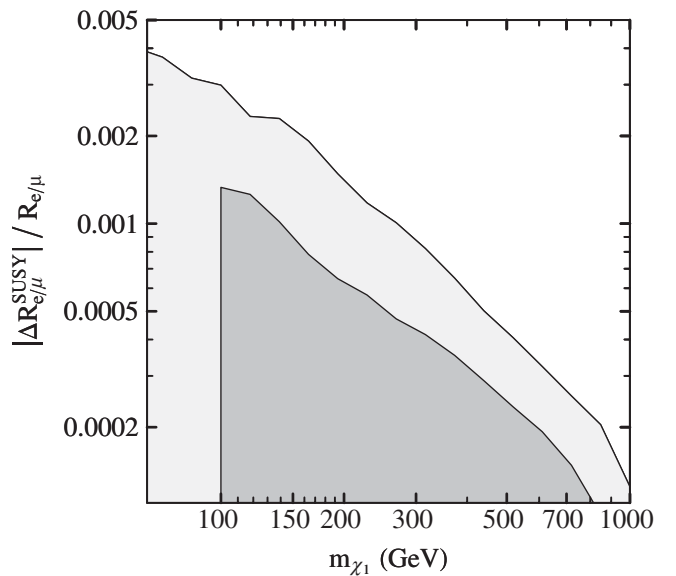

FIG. 8. $\Delta R_{e/\mu}^{\text{SUSY}}$ versus $m_{\chi 1}$, the mass of the lightest chargino. The dark and light regions denote the regions of MSSM parameter space consistent and inconsistent, respectively, with the LEP II bound.
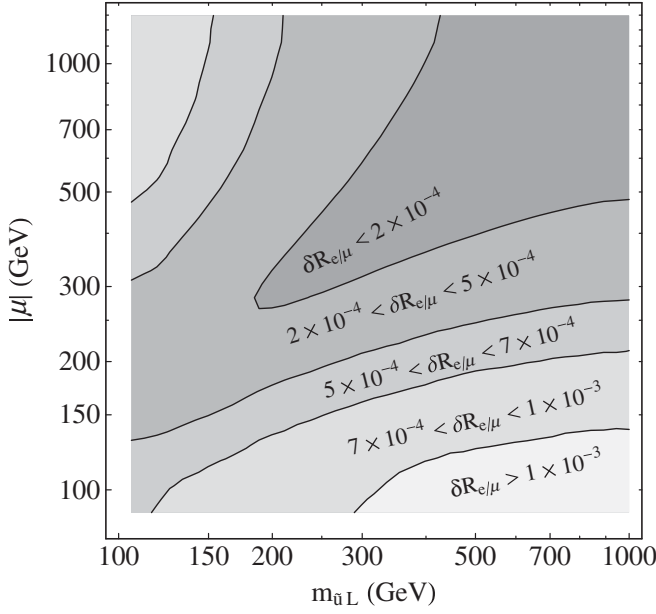
FIG. 9. Contours indicate the largest values of $\delta R_{e/\mu}^{\rm SUSY}$ obtained by our numerical parameter scan (26), as a function of $|\mu|$ and $m_{\tilde{u}_L}$. The solid shaded regions correspond to the largest values of $\delta R_{e/\mu}^{\rm SUSY}$ within the ranges indicated. All values of $\delta R_{e/\mu}^{\rm SUSY}$ correspond to parameter points which satisfy the LEP II bound.

sleptons are degenerate, then $\Delta R_{e/\mu}^{\rm SUSY}$ vanishes. Assuming the LEP II bound, in order for a deviation in $R_{e/\mu}$ to match the target precision at upcoming experiments, we must have

$$\delta R_{e/\mu} \equiv |\Delta R_{e/\mu}^{\rm SUSY}/R_{e/\mu}| \gtrsim 0.0005, \qquad (28)$$

and thus $m_{\tilde{e}_L}/m_{\tilde{\mu}_L} \gtrsim 2$ or $m_{\tilde{\mu}_L}/m_{\tilde{e}_L} \gtrsim 2$. (This result is consistent with an earlier analysis [22], where the authors conclude that $\Delta R_{e/\mu}^{\rm SUSY}$ would be unobservably small if m_{e_L} and m_{μ_L} differ by less than 10%.)

In Fig. 7, we show $\Delta R_{e/\mu}^{\rm SUSY}/R_{e/\mu}$ as a function of $\mathrm{Min}[m_{\tilde{e}_L}, m_{\tilde{\mu}_L}]$, the mass lightest first or second generation slepton. If the lighter slepton is extremely heavy, then both heavy sleptons decouple, causing $\Delta R_{e/\mu}^{\rm SUSY}$ to vanish. Assuming the LEP II bound, to satisfy (28), we must have $m_{\tilde{e}_L} \lesssim 300$ GeV or $m_{\tilde{\mu}_L} \lesssim 300$ GeV.

In Fig. 8, we show $\Delta R_{e/\mu}^{\rm SUSY}/R_{e/\mu}$ as a function of $m_{\chi 1}$, the lightest chargino mass. If $m_{\chi 1}$ is large, $\Delta R_{e/\mu}^{\rm SUSY}$ vanishes because M_2 must be large as well, suppressing Δ_B and forcing Δ_V and Δ_L to sum to the flavor-independent constant discussed above. Assuming the LEP II bound, to satisfy (28), we must have $m_{\chi 1} \lesssim 250$ GeV.

Finally, we illustrate the interplay between $\Delta_V + \Delta_L$ and Δ_B by considering $\delta R_{e/\mu}^{\rm SUSY}$ as a function of $|\mu|$ and $m_{\tilde{u}_L}$. In Fig. 9, we show the largest values of $\delta R_{e/\mu}^{\rm SUSY}$ obtained in our numerical parameter scan, restricting to

parameter points which satisfy the LEP II bound. The solid shaded areas correspond to regions of the $|\mu|$-$m_{\tilde{u}_L}$ plane where the largest value of $\delta R_{e/\mu}^{\rm SUSY}$ lies within the indicated ranges. It is clear that $\delta R_{e/\mu}^{\rm SUSY}$ can be largest in the regions where either (1) μ is small, $m_{\tilde{u}_L}$ is large, and the largest contributions to $\Delta R_{e/\mu}^{\rm SUSY}$ are from $\Delta_V + \Delta_L$, or (2) μ is large, $m_{\tilde{u}_L}$ is small, and the largest contribution to $\Delta R_{e/\mu}^{\rm SUSY}$ is from Δ_B. If both μ and $m_{\tilde{u}_L}$ are light, then $\Delta R_{e/\mu}^{\rm SUSY}$ can still be very small due to cancellations, even though both $\Delta_V + \Delta_L$ and Δ_B contributions are large individually. More precisely, to satisfy (28), we need either $\mu \lesssim 250$ GeV, or $\mu \gtrsim 300$ GeV and $m_{\tilde{u}_L} \lesssim 200$ GeV.

III. CONTRIBUTIONS FROM R-PARITY-VIOLATING PROCESSES

In the presence of RPV interactions, tree-level exchanges of sfermions (shown in Fig. 10), lead to violations of lepton universality and nonvanishing effects in $R_{e/\mu}$. The magnitude of these tree-level contributions is governed by both the sfermion masses and by the parameters λ'_{11k} and λ'_{21k} that are the coefficients in RPV interactions:

$$\mathcal{L}_{RPV,\Delta L=1} = \lambda'_{ijk} L_i Q_j \tilde{d}_k^\dagger + \dots \qquad (29)$$

Defining [29,30]

$$\Delta'_{ijk}(\tilde{f}) = \frac{|\lambda'_{ijk}|^2}{4\sqrt{2} G_\mu m_{\tilde{f}}^2} \geq 0, \qquad (30)$$

contributions to $R_{e/\mu}$ from RPV interactions are

$$\frac{\Delta R_{e/\mu}^{RPV}}{R_{e/\mu}^{SM}} = 2\Delta'_{11k} - 2\Delta'_{21k}. \qquad (31)$$

Note that RPV contribution to the muon lifetime (and, thus, the Fermi constant G_μ) cancels in $R_{e/\mu}$, therefore does not enter the expression.

The quantities Δ'_{ijk} etc. are constrained by existing precision measurements and rare decays. A summary of the low-energy constraints is given in Table III of Ref. [10], which includes tests of CKM unitarity (primarily through RPV effects in superallowed nuclear β-decay that yields a precise value of $|V_{ud}|$ [31]), atomic parity-violating (PV) measurements of the cesium weak charge $Q_W^{\rm Cs}$ [32], the
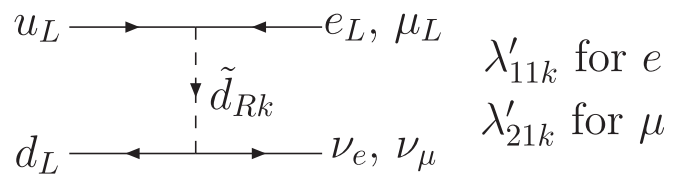

FIG. 10. Tree-level RPV contributions to $R_{e/\mu}$.

ratio $R_{e/\mu}$ itself [15,16], a comparison of the Fermi constant G_μ with the appropriate combination of α, m_Z, and $\sin^2\theta_W$ [33], and the electron weak charge determined from SLAC E158 measurement of parity-violating Møller scattering [34].

In Fig. 11 we show the present 95% C.L. constraints on the quantities Δ'_{11k} and Δ'_{21k} obtained from the aforementioned observables [interior of the blue (dark gray) curve]. Since the Δ'_{ijk} are positive semidefinite quantities, only the region of the contour in the upper right-hand quadrant are shown. The green (light gray) curve indicates the possible implication of a future measurement of the proton weak charge planned at Jefferson Lab [35], assuming agreement with the standard model prediction for this quantity and the anticipated experimental uncertainty. The dashed red (gray) curve shows the possible impact of future measurements of $R_{e/\mu}$, assuming agreement with the present central value but an overall error reduced to the level anticipated in Ref. [18]; with the error anticipated in Ref. [19] the width of the band would be a factor of 2 smaller than shown.

Two general observations emerge from Fig. 11. First, given the present constraints, values of Δ'_{21k} and Δ'_{11k} differing substantially from zero are allowed. For values of these quantities inside the blue (dark gray) contour, $\Delta R^{SUSY}_{e/\mu}$ could differ from zero by up to 5 standard deviations for the error anticipated in Ref. [18]. Such RPV effects could, thus, be considerably larger than the SUSY
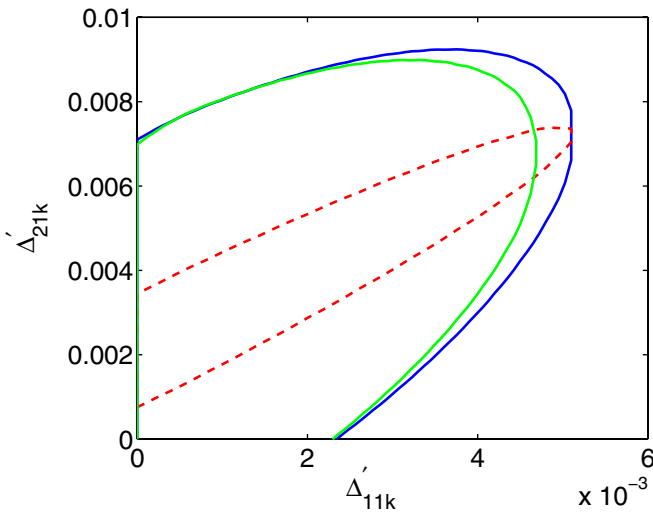

FIG. 11 (color online). Present 95% C.L. constraints on RPV parameters Δ'_{j1k}, $j = 1, 2$ that enter $R_{e/\mu}$ obtained from a fit to precision electroweak observables. Interior of the dark blue (dark gray) contour corresponds to the fit using the current value of $\Delta R_{e/\mu}/R^{SM}_{e/\mu}$ [15,16], while the dashed red (gray) contour corresponds to the fit using the future expected experimental precision [18], assuming the same central value. The light green (light gray) curve indicates prospective impact of a future measurement of the proton weak charge at Jefferson Lab [35].

loop corrections discussed above. On the other hand, agreement of $R_{e/\mu}$ with the SM would lead to considerable tightening of the constraints on this scenario, particularly in the case of Δ'_{21k}, which is currently constrained only by $R_{e/\mu}$ and deep inelastic ν $(\bar{\nu})$ scattering [36].

The presence of RPV interactions would have significant implications for both neutrino physics and cosmology. It has long been known, for example, that the existence of $\Delta L = \pm 1$ interactions—such as those that could enter $R_{e/\mu}$—will induce a Majorana neutrino mass [37], while the presence of nonvanishing RPV couplings would imply that the lightest supersymmetric particle is unstable and, therefore, not a viable candidate for cold dark matter. The future measurements of $R_{e/\mu}$ could lead to substantially tighter constraints on these possibilities or uncover a possible indication of RPV effects. In addition, we note that the present uncertainty associated with RPV effects entering the $\pi_{\mu2}$ decay rate would affect the value of F_π at a level of about half the theoretical SM uncertainty as estimated by Ref. [12].

IV. CONCLUSIONS

Given the prospect of two new studies of lepton universality in $\pi_{\ell2}$ decays [18,19] with experimental errors that are substantially smaller than for existing measurements and possibly approaching the 5×10^{-4} level, an analysis of the possible implications for supersymmetry is a timely exercise. In this study, we have considered SUSY effects on the ratio $R_{e/\mu}$ in the MSSM both with and without R-parity violation. Our results indicate that in the R-parity conserving case, effects from SUSY loops can be of order the planned experimental error, in particular, limited regions of the MSSM parameter space. Specifically, we find that a deviation in $R_{e/\mu}$ due to the MSSM at the level of

$$0.0005 \lesssim \left| \frac{\Delta R^{SUSY}_{e/\mu}}{R_{e/\mu}} \right| \lesssim 0.001, \qquad (32)$$

implies (1) the lightest chargino χ_1 is sufficiently light

$$m_{\chi1} \lesssim 250 \text{ GeV},$$

(2) the left-handed selectron $\tilde{e}_L$ and smuon $\tilde{\mu}_L$ are highly nondegenerate:

$$\frac{m_{\tilde{e}_L}}{m_{\tilde{\mu}_L}} \gtrsim 2 \quad \text{or} \quad \frac{m_{\tilde{e}_L}}{m_{\tilde{\mu}_L}} \lesssim \frac{1}{2},$$

(3) at least one of $\tilde{e}_L$ or $\tilde{\mu}_L$ must be light, such that

$$m_{\tilde{e}_L} \lesssim 300 \text{ GeV} \quad \text{or} \quad m_{\tilde{\mu}_L} \lesssim 300 \text{ GeV},$$

and (4) the Higgsino mass parameter μ and left-handed up squark mass $m_{\tilde{u}_L}$ satisfy either

$$|\mu| \lesssim 250 \text{ GeV}$$

or

$$|\mu| \gtrsim 300 \text{ GeV}, \qquad m_{\tilde{\mu}_L} \lesssim 200 \text{ GeV}.$$

Under these conditions, the magnitude $\Delta R_{e/\mu}^{\text{SUSY}}$ may fall within the sensitivity of the new $R_{e/\mu}$ measurements.

In conventional scenarios for SUSY-breaking mediation, one expects the left-handed slepton masses to be comparable, implying no substantial corrections to SM predictions for $R_{e/\mu}$. Significant reductions in both experimental error and theoretical, hadronic physics uncertainties in $R_{e/\mu}^{SM}$ would be needed to make this ratio an effective probe of the superpartner spectrum.

On the other hand, constraints from existing precision electroweak measurements leave considerable latitude for observable effects from tree-level superpartner exchange in the presence of RPV interactions. The existence of such effects would have important consequences for both neutrino physics and cosmology, as the presence of the $\Delta L = 0$ RPV interactions would induce a Majorana mass term for the neutrino and allow the lightest superpartner to decay to SM particles too rapidly to make it a viable dark matter candidate. Agreement between the results of the new $R_{e/\mu}$

measurements with $R_{e/\mu}^{SM}$ could yield significant new constraints on these possibilities.

APPENDIX: GENERAL RADIATIVE CORRECTIONS IN THE MSSM

The MSSM Lagrangian and Feynman rules [38] are expressed in terms of chargino and neutralino mixing matrices $Z_\pm$ and Z_N, respectively, which diagonalize the superpartner mass matrices, defined as follows. The four neutralino mass eigenstates χ_j^0 are related to the gauge eigenstates $\psi^0 \equiv (\tilde{B}, \tilde{W}^3, \tilde{H}_d^0, \tilde{H}_u^0)$ by

$$\psi_i^0 = Z_N^{ij}\chi_j^0, \tag{A1}$$

where

$$Z_N^T \begin{pmatrix} M_1 & 0 & -c_\beta s_W m_Z & s_\beta s_W m_Z \\ 0 & M_2 & c_\beta c_W m_Z & -s_\beta c_W m_Z \\ -c_\beta s_W m_Z & c_\beta c_W m_Z & 0 & -\mu \\ s_\beta s_W m_Z & -s_\beta c_W m_Z & -\mu & 0 \end{pmatrix} Z_N = \begin{pmatrix} m_{\chi_1^0} & 0 & 0 & 0 \\ 0 & m_{\chi_2^0} & 0 & 0 \\ 0 & 0 & m_{\chi_3^0} & 0 \\ 0 & 0 & 0 & m_{\chi_4^0} \end{pmatrix} \tag{A2}$$

is the diagonalized neutralino mass matrix. The chargino mass eigenstates $\chi_i^\pm$ are related to the gauge eigenstates $\psi^+ \equiv (\tilde{W}^+, \tilde{H}_u^+)$ and $\psi^- \equiv (\tilde{W}^-, \tilde{H}_d^-)$ by

$$\psi_i^\pm = Z_\pm^{ij}\chi_j^\pm, \tag{A3}$$

where

$$Z_-^T \begin{pmatrix} M_2 & \sqrt{2}s_\beta m_W \\ \sqrt{2}c_\beta m_W & \mu \end{pmatrix} \quad Z_+ = \begin{pmatrix} m_{\chi_1} & 0 \\ 0 & m_{\chi_2} \end{pmatrix} \tag{A4}$$

is the diagonalized chargino mass matrix. We note that the off-diagonal elements which mix gauginos and Higgsinos stem solely from electroweak symmetry breaking.

The charged slepton mass eigenstates $\tilde{L}_i$ are related to the gauge eigenstates $\tilde{\ell} \equiv (\tilde{e}_L, \tilde{\mu}_L, \tilde{\tau}_L, \tilde{e}_R, \tilde{\mu}_R, \tilde{\tau}_R)$ by

$$\tilde{\ell}_i = Z_L^{ij}\tilde{L}_j, \tag{A5}$$

where

$$Z_L^\dagger \mathbf{M}_{\tilde{\ell}}^2 Z_L = \begin{pmatrix} m_{\tilde{L}_1}^2 & & 0 \\ & \ddots & \\ 0 & & m_{\tilde{L}_6}^2 \end{pmatrix} \tag{A6}$$

is the diagonalized slepton mass matrix. There are two classes of off-diagonal elements in $\mathbf{M}_{\tilde{\ell}}^2$ which can contribute to slepton mixing: mixing between flavors and mixing between left- and right-handed components of a given flavor, both of which arise through SUSY-breaking terms. (Left-right mixing due to SUSY-preserving terms will be suppressed by $m_\ell/m_{\tilde{\ell}}$ and is irrelevant for the first two generations.)

Similarly, up-type squarks, down-type squarks, and sneutrinos have mixing matrices Z_U, Z_D, and Z_ν, respectively, defined identically to Z_L—except for the fact that there are no right-handed sneutrinos in the MSSM and thus there are only three sneutrino mass eigenstates.

There are three types of contributions to $\Delta R_{e/\mu}^{\text{SUSY}}$ in the MSSM: external leg, vertex, and box graph radiative corrections. The leptonic external leg corrections [Fig. 1(b)] are

$$\begin{aligned} \Delta_L^{(i)} = &-\frac{\alpha}{16\pi s_W^2}(|Z_N^{1j}t_W - Z_N^{2j}|^2 B(m_{\chi_j^0}, m_{\tilde{\nu}_i}) \\ &+ 2|Z_-^{1k}|^2 B(m_{\chi_k}, m_{\tilde{L}_i}) + |Z_N^{1j}t_W + Z_N^{2j}|^2 B(m_{\chi_j^0}, m_{\tilde{L}_i}) \\ &+ 2|Z_+^{1k}|^2 B(m_{\chi_k}, m_{\tilde{\nu}_i})), \end{aligned} \tag{A7}$$

where the loop function is [39]

$$B(m_1, m_2) = \int_0^1 dx\, x \ln\left(\frac{M^2}{m_1^2(1-x) + m_2^2 x}\right).$$

The leptonic vertex corrections [Fig. 1(c)] are

$$
\begin{aligned}
\Delta_V^{(I)} = \frac{\alpha}{8\pi s_W^2} \Bigg(& (Z_N^{1j} t_W + Z_N^{2j})(Z_N^{1j*} t_W - Z_N^{2j*}) C_2(m_{\tilde{\nu}_i}, m_{\chi_j^0}, m_{\tilde{L}_i}) \\
& + 2(Z_N^{2j*} - t_W Z_N^{1j*}) Z_+^{1k} \left[\left(Z_N^{2j} Z_+^{1k} - \frac{1}{\sqrt{2}} Z_N^{4j} Z_+^{2k} \right) C_2(m_{\chi_j^0}, m_{\tilde{\nu}_i}, m_{\chi_k}) \right. \\
& \left. + \left(Z_N^{2j*} Z_-^{1k} + \frac{1}{\sqrt{2}} Z_N^{3j*} Z_-^{2k} \right) m_{\chi_j^0} m_{\chi_k} C_1(m_{\chi_j^0}, m_{\tilde{\nu}_i}, m_{\chi_k}) \right] \\
& + 2(Z_N^{2j} + t_W Z_N^{1j}) Z_-^{1k} \left[\left(Z_N^{2j*} Z_-^{1k} + \frac{1}{\sqrt{2}} Z_N^{3j*} Z_-^{2k} \right) C_2(m_{\chi_k}, m_{\tilde{L}_i}, m_{\chi_j^0}) \right. \\
& \left. + \left(Z_N^{2j} Z_+^{1k} - \frac{1}{\sqrt{2}} Z_N^{4j} Z_+^{2k} \right) m_{\chi_j^0} m_{\chi_k} C_1(m_{\chi_k}, m_{\tilde{L}_i}, m_{\chi_j^0}) \right] \Bigg),
\end{aligned}
\tag{A8}
$$

with loop functions

$$C_1(m_1, m_2, m_3) = \int_0^1 dx\,dy \frac{1}{m_1^2 x + m_2^2 y + m_3^2(1 - x - y)}$$

$$C_2(m_1, m_2, m_3) = \int_0^1 dx\,dy \ln\left(\frac{M^2}{m_1^2 x + m_2^2 y + m_3^2(1 - x - y)}\right).$$

The corrections from box graphs [Fig. 1(d)] are

$$
\begin{aligned}
\Delta_B^{(I)} = \frac{\alpha m_W^2}{8\pi s_W^2} \Bigg(& |Z_-^{1k}|^2 (Z_N^{2m*} + t_W Z_N^{1m*}) \left(Z_N^{2m} - \frac{1}{3} t_W Z_N^{1m} \right) D_1(m_{\chi_m^0}, m_{\tilde{d}_L}, m_{\chi_k}, m_{\tilde{L}_i}) \\
& + |Z_+^{1j}|^2 (Z_N^{2m} - t_W Z_N^{1m}) \left(Z_N^{2m*} + \frac{1}{3} t_W Z_N^{1m*} \right) D_1(m_{\chi_j}, m_{\tilde{u}_L}, m_{\chi_m^0}, m_{\tilde{\nu}_i}) \\
& + Z_-^{1j} Z_+^{1j} (Z_N^{2m} - t_W Z_N^{1m}) \left(Z_N^{2m} - \frac{1}{3} t_W Z_N^{1m} \right) m_{\chi_m^0} m_{\chi_j} D_2(m_{\chi_m^0}, m_{\tilde{d}_L}, m_{\chi_j}, m_{\tilde{\nu}_i}) \\
& + Z_-^{1k} Z_+^{1k} (Z_N^{2m*} + t_W Z_N^{1m*}) \left(Z_N^{2m*} + \frac{1}{3} t_W Z_N^{1m*} \right) m_{\chi_m^0} m_{\chi_k} D_2(m_{\chi_k}, m_{\tilde{u}_L}, m_{\chi_m^0}, m_{\tilde{L}_i}) \Bigg),
\end{aligned}
\tag{A9}
$$

with loop functions

$$D_n(m_1, m_2, m_3, m_4) = \int_0^1 dx\,dy\,dz \frac{1}{[m_1^2 x + m_2^2 y + m_3^2 z + m_4^2(1 - x - y - z)]^n}.$$

In formulas (A7)–(A9), $I = 1$ corresponds to $\pi \to e\nu_e$ and $I = 2$ corresponds to $\pi \to \mu\nu_\mu$. All other indices are summed over. We use $\overline{\text{DR}}$ renormalization at scale M. We have defined $t_W \equiv \tan\theta_W$ and $s_W \equiv \sin\theta_W$. We have neglected terms proportional to either Yukawa couplings or external momenta [which will be suppressed by $\mathcal{O}(m_\pi/M_{\text{SUSY}})$]. Finally, the SUSY contribution to $R_{e/\mu}$ is

$$\frac{\Delta R_{e/\mu}^{\text{SUSY}}}{R_{e/\mu}} = 2\,\text{Re}[\Delta_V^{(1)} - \Delta_V^{(2)} + \Delta_L^{(1)} - \Delta_L^{(2)} + \Delta_B^{(1)} - \Delta_B^{(2)}]. \tag{A10}$$

In addition, the following are some useful formulas needed to show the cancellations of vertex and leg corrections in the limit of no superpartner mixing:

$$C_2(m_1, m_2, m_1) = B(m_2, m_1) \qquad 2m_1^2 C_1(m_1, m_2, m_1) - 2B(m_1, m_2) + 2B(m_2, m_1) = 1.$$

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