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Simultaneous Extraction of the Fermi constant and PMNS matrix elements in the presence of a fourth generation

Several recent studies performed on constraints of a fourth generation of quarks and leptons suffer from the ad-hoc assumption that 3×3 unitarity holds for the first three generations in the neutrino sector. Only under this assumption one is able to determine the Fermi constant G_F from the muon lifetime measurement with the claimed precision of $G_F = 1.16637(1) \times 10^{-5} \text{ GeV}^{-2}$. We study how well G_F can be extracted within the framework of four generations from leptonic and radiative μ and τ decays, as well as from $K_{\ell 3}$ decays and leptonic decays of charged pions, and we discuss the role of lepton universality tests in this context. We emphasize that constraints on a fourth generation from quark and lepton flavour observables and from electroweak precision observables can only be obtained in a consistent way if these three sectors are considered simultaneously. In the combined fit to leptonic and radiative μ and τ decays, $K_{\ell 3}$ decays and leptonic decays of charged pions we find a p-value of 2.6 % for the fourth generation matrix element $|U_{e4}| = 0$ of the neutrino mixing matrix.

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1 Introduction

After a rather long period of low activity concerning studies of an extension of the three generation Standard Model (3SM) to a fourth generation Standard Model (4SM) a revival of the topic is observed in recent years in the literature. The possible role of a fourth generation in explaining some interesting hints for deviations from the Standard Model (SM) in flavour physics has been discussed in several papers [1–9]. The electroweak precision fit with respect to a fourth generation has been reconsidered as well [10–12]. This triggered more intensive work on the fourth generation, e.g. on constraints from the electroweak precision fit [13] taking into account also the dependence on CKM [14] matrix elements in the T-parameter, or on the role in the anomalous magnetic moment of the muon [15].

The above mentioned studies rely on the implicit, but often not recognized, assumption that the extraction of a central parameter in the SM is unaffected by the presence of a fourth generation: the value of the Fermi constant G_F extracted from the lifetime measurement of muons. The assumption made is that the 3SM neutrino mixing (PMNS [16]) matrix fulfills exact 3×3 unitarity already in the first three generations. Under this assumption the PMNS matrix elements U_{e4} , $U_{\mu 4}$ and $U_{\tau 4}$ are all zero and the relative uncertainty of the extracted G_F value (10^{-5}) is so small that all these studies simply consider G_F as a parameter without uncertainties. One might naively expect small or even tiny PMNS elements for the mixing between a fourth generation neutrino, which must be heavy to escape the bound from the Z -resonance width [17], and the light neutrinos of the first three generations. Nevertheless, the assumption of zero mixing elements is certainly not justified, in particular, since we neither understand how family replication works, nor how the values of the mixing matrix elements are fixed by Nature.

In this paper, we emphasize that taking G_F as a constant parameter is not *a-priori* justified and that the precision on G_F extracted from leptonic μ and τ decays and from $K_{\ell 3}$ decays and leptonic decays of charged pions, once this assumption is abandoned, is only of order 50 % despite the fact that lepton universality is tested to a very good precision in these decays. When adding the information from the search limits for the radiative decays $\mu \rightarrow e\gamma$, $\tau \rightarrow e\gamma$ and $\tau \rightarrow \mu\gamma$ we show that the precision on G_F is significantly improved. Nevertheless, the precision of the extracted G_F value is still about two orders of magnitude worse than in the 3SM case. As a consequence, statements made about a fourth generation scenario with respect to PMNS and/or CKM quark mixing matrix elements as well as concerning fourth generation quark and lepton mass differences and Higgs mass bounds from an electroweak precision fit like in [10–12] possibly suffer from this source of uncertainty that has not been considered in these kind of studies.

Studies of a non-unitary PMNS and/or CKM matrix are not completely new. Lepton or quark universality violations and possible effects on the extraction of G_F has been already considered in several papers [19–24]. Mixing between ordinary and “exotic” fermions as

an extension of the 3SM has been studied in quite some quantitative detail in Ref. [25]. Ref. [26] focused on the aspects of the non-decoupling quadratic dependence on the heavy neutrino mass in lepton flavour violating processes like $\mu \rightarrow e\gamma$ and $\mu \rightarrow ee^+e^-$. In Ref. [27] a detailed analysis of possible violations of unitarity of the 3×3 PMNS matrix has been presented under the assumption of “Minimal Unitarity Violation” (MVU): only three light neutrinos are considered and New Physics is introduced in the neutrino sector only. The authors of Ref. [27] considered W decays, invisible Z decays, leptonic μ and τ decays as well as leptonic π decays together with radiative μ and τ decays and combined this information for the first time with neutrino oscillation data. Refinements of this analysis can be found in Ref. [28] where also G_F has been constrained using the unitarity constraint in the first row of the CKM matrix while replacing the constraint from invisible Z decays. Effects of a non-unitary PMNS matrix on CP violation in neutrino oscillation observables within the MVU framework have been studied in Refs. [29] and [30].

In our paper we explicitly study constraints when allowing for an additional fourth lepton and quark generation. Our analysis is contained in the more general analysis of Refs. [27] and [28] but more specific since a smaller number of new parameters is needed to account for unitarity violations. The only case where the analysis of Ref. [27] is more specific than ours is the usage of invisible Z decays where the authors of Ref. [27] only take into account those radiative corrections which originate from the 3SM.

The numerical studies presented in our paper have been performed using the software package CKMfitter [18] where the four generation PMNS and CKM matrix have been implemented using the Botella-Chau parametrisation [31].

2 Extraction of G_F from leptonic μ and τ decays

Let us first consider the μ decay which is dominated by the final state $\mu^- \rightarrow e^- \bar{\nu}_e \nu_\mu$. In the μ -lifetime measurement neither the neutrinos in the final state are detected nor is it possible to decide for a given event which neutrino mass eigenstate has been produced. The predicted muon decay-rate $\Gamma(\mu \rightarrow all) = \Gamma(\mu^- \rightarrow e^- \bar{\nu}_e \nu_\mu) + \Gamma(\mu^- \rightarrow e^- \bar{\nu}_e \nu_\mu + \gamma) + \dots$, which takes into account electroweak corrections [32–35], is given by

$$\Gamma(\mu^- \rightarrow all) = \frac{G_F^2 m_\mu^5}{192\pi^3} \cdot PS(m_e, m_\mu) \cdot \left[\left(1 - \frac{\alpha(m_\mu^2)}{2\pi} \left(\pi^2 - \frac{25}{4}\right) + C_2 \frac{\alpha^2(m_\mu^2)}{\pi^2}\right) \left(1 + \frac{3m_\mu^2}{5m_W^2}\right) \right] \cdot \sum_{i=1,2,3} |U_{ei}|^2 \sum_{j=1,2,3} |U_{\mu j}|^2, \quad (1)$$

where G_F is the Fermi constant to be determined, $m_\mu = (105.658367 \pm 0.000004)$ MeV [38] and $m_e = (0.510998910 \pm 0.000000013)$ MeV [38] is the muon, respectively, the electron mass, $m_W = (80.398 \pm 0.025)$ MeV [38] is the W -boson mass, $\alpha(m_\mu^2)$ is the fine structure constant at the scale m_μ^2 calculated using Ref. [39], $C_2 = (6.700 \pm 0.002)$ is the coefficient of the $O(\alpha^2)$ radiative corrections [35], and

$$PS(m_l, m_i) = 1 - 8 \left(\frac{m_l}{m_i}\right)^2 - 12 \left(\frac{m_l}{m_i}\right)^4 \log \left(\frac{m_l}{m_i}\right)^2 + 8 \left(\frac{m_l}{m_i}\right)^6 - \left(\frac{m_l}{m_i}\right)^8 \quad (2)$$

is a factor that accounts for the correction of the phase space due to finite masses in the final state. Here, it is assumed that only the electron has a significant mass while the neutrino masses are neglected in the phase space correction factor. The world average of the muon life $\tau_\mu = \Gamma^{-1}(\mu^- \rightarrow \text{all})$ reads $\tau_\mu = (2.197034 \pm 0.000021) \cdot 10^{-6}$ s [38] where the uncertainty has been halved recently by measurements from the MuLan [36] and the FAST [37] collaborations.

The U_{ei} and $U_{\mu j}$ are the PMNS matrix elements for the three light neutrino mass states of the first three generations. In the 3SM the sums $\sum_{i=1,2,3} |U_{ei}|^2$ and $\sum_{j=1,2,3} |U_{\mu j}|^2$ are exactly equal to 1 due to 3×3 unitarity. In the 4SM the fourth generation neutrino must have a mass of at least about half of the Z-boson mass as the number of light neutrino flavours from measuring the partial widths of visible final states and the total width of the Z-resonance at LEP does exclude a light fourth neutrino flavour [17]. Hence, the additional fourth generation neutrino is not kinematically accessible in μ -decays. Although the same formula for the $\mu^- \rightarrow e^- \bar{\nu}_e \nu_\mu$ applies, the sums $\sum_{i=1,2,3} |U_{ei}|^2$ and $\sum_{j=1,2,3} |U_{\mu j}|^2$ do not necessarily add up to 1 any more. In a fourth generation scenario one can write $\sum_{i=1,2,3} |U_{ei}|^2 = 1 - |U_{e4}|^2$ and $\sum_{j=1,2,3} |U_{\mu j}|^2 = 1 - |U_{\mu 4}|^2$ thanks to 4×4 unitarity of the fourth generation PMNS matrix.

Similar in line the partial rates for the leptonic τ decays including electroweak corrections at leading logarithm are given by [34, 62]

$$\begin{aligned} \Gamma(\tau^- \rightarrow e^- \bar{\nu}_e \nu_\tau + (\gamma)) &= \frac{G_F^2 m_\tau^5}{192\pi^3} \cdot PS(m_e, m_\tau) \cdot \left[\left(1 - \frac{\alpha(m_\tau^2)}{2\pi} (\pi^2 - \frac{25}{4})\right) \left(1 + \frac{3m_\tau^2}{5m_W^2}\right) \right] \\ &\cdot (1 - |U_{e4}|^2)(1 - |U_{\tau 4}|^2), \end{aligned} \quad (3)$$

and

$$\begin{aligned} \Gamma(\tau^- \rightarrow \mu^- \bar{\nu}_\mu \nu_\tau + (\gamma)) &= \frac{G_F^2 m_\tau^5}{192\pi^3} \cdot PS(m_\mu, m_\tau) \cdot \left[\left(1 - \frac{\alpha(m_\tau^2)}{2\pi} (\pi^2 - \frac{25}{4})\right) \left(1 + \frac{3m_\tau^2}{5m_W^2}\right) \right] \\ &\cdot (1 - |U_{\mu 4}|^2)(1 - |U_{\tau 4}|^2). \end{aligned} \quad (4)$$

These decay rates are experimentally determined from the measured τ lifetime $\tau_\tau = (290.6 \pm 1.0) \cdot 10^{-15}$ s [38] and the corresponding measured branching fractions as $\Gamma(\tau^- \rightarrow e^- \bar{\nu}_e \nu_\tau + (\gamma)) = BF(\tau^- \rightarrow e^- \bar{\nu}_e \nu_\tau + (\gamma))/\tau_\tau$ and $\Gamma(\tau^- \rightarrow \mu^- \bar{\nu}_\mu \nu_\tau + (\gamma)) = BF(\tau^- \rightarrow \mu^- \bar{\nu}_\mu \nu_\tau + (\gamma))/\tau_\tau$. The dominant uncertainties in the prediction of these branching fractions are induced by the τ lifetime and the τ mass $m_\tau = (1776.84 \pm 0.17)$ MeV [38].

One might be tempted to assume that the well-measured lepton universality, when comparing the values for the Fermi constant extracted from the experimentally determined values of $\Gamma(\mu^- \rightarrow e^- \bar{\nu}_e \nu_\mu)$, $\Gamma(\tau^- \rightarrow e^- \bar{\nu}_e \nu_\tau)$, and $\Gamma(\tau^- \rightarrow \mu^- \bar{\nu}_\mu \nu_\tau)$, should constrain the elements U_{e4} , $U_{\mu 4}$ and $U_{\tau 4}$ to be close to zero. However, this is not the case since these measurements can only constrain the products $G_F^2(1 - |U_{\mu 4}|^2)(1 - |U_{e4}|^2)$, $G_F^2(1 - |U_{\tau 4}|^2)(1 - |U_{e4}|^2)$, and $G_F^2(1 - |U_{\mu 4}|^2)(1 - |U_{\tau 4}|^2)$. As a consequence, the observed lepton universality constrains only the ratios $|U_{e4}|/|U_{\mu 4}|$, $|U_{\tau 4}|/|U_{e4}|$, and $|U_{\tau 4}|/|U_{\mu 4}|$ to stay close to 1. E.g., for sufficiently large $|U_{e4}|$ and $|U_{\mu 4}|$ values one has to a very good approximation $(1 - |U_{e4}|^2)/(1 - |U_{\mu 4}|^2) = 1$ from which follows $|U_{e4}| = |U_{\mu 4}|$. As a result, lepton universality alone allows surprisingly large values for U_{e4} , $U_{\mu 4}$ and $U_{\tau 4}$ and in this case these elements show a very strong correlation. The maximally allowed values are easily determined

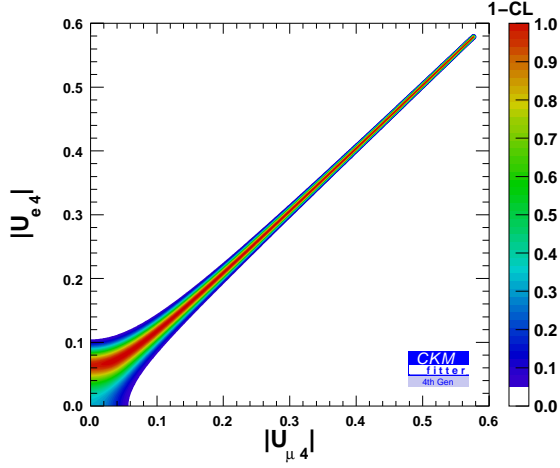
without any sophisticated numerical analysis: unitarity requires $|U_{e4}|^2 + |U_{\mu4}|^2 + |U_{\tau4}|^2 \leq 1$ while perfect universality results in $|U_{e4}| = |U_{\mu4}| = |U_{\tau4}|$. As a consequence, lepton universality is perfectly compatible with $|U_{e4}|, |U_{\mu4}|, |U_{\tau4}| \leq 1/\sqrt{3} \approx 0.577$. In this worst case limit, G_F could have a value that is larger by a factor of 1.5 with respect to the value extracted from the 3SM scenario.

In what follows we denote the value of G_F extracted from the muon lifetime measurement when assuming 3×3 unitarity of the PMNS matrix for the first three neutrino generations as $G_F^{3SM} = 1.16637(1) \times 10^{-5} \text{ GeV}^{-2}$ by setting $|U_{e4}| = |U_{\mu4}| = |U_{\tau4}| = 0$ while in the general four generation case it is denoted as G_F^{4SM} .

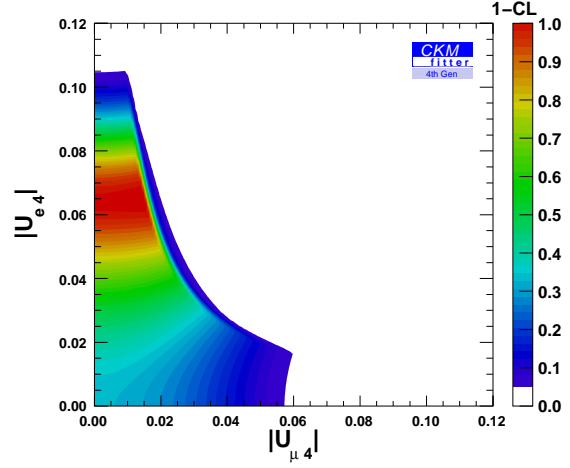
Figs. 1(a), 1(c), and 1(e) show the actual constraints on $|U_{e4}|$ versus $|U_{\mu4}|$, $|U_{\tau4}|$ versus $|U_{\mu4}|$, and $|U_{e4}|$ versus $|U_{\tau4}|$ when using the input values $BF(\tau \rightarrow e\nu\bar{\nu}) = 0.1785 \pm 0.0005$ [38] and $BF(\tau \rightarrow \mu\nu\bar{\nu}) = 0.1736 \pm 0.0005$ [38] measured at LEP I, $BF(\tau \rightarrow \mu\nu\bar{\nu})/BF(\tau \rightarrow e\nu\bar{\nu}) = 0.9796 \pm 0.0016 \pm 0.0036$ as recently measured by BABAR [40], and the μ lifetime $\tau_\mu = (2.197034 \pm 0.000021) \times 10^{-6} \text{ s}$ [38]. In the numerical analysis we explicitly take into account the correlation coefficient of -13% between the LEP I measurements of $BF(\tau \rightarrow e\nu\bar{\nu})$ and $BF(\tau \rightarrow \mu\nu\bar{\nu})$ as quoted in [38]. We stress that the uncertainties from τ_τ and m_τ cancel in the constraint $|U_{e4}|$ versus $|U_{\mu4}|$, but not in $|U_{\tau4}|$ versus $|U_{\mu4}|$ and $|U_{e4}|$ versus $|U_{\tau4}|$. One clearly sees the high correlation induced by the experimentally well-established lepton universality with the upper limit being close to the theoretical limit discussed just before. The observed correlation starts to vanish for values below about 0.1 due to the finite experimental precision. In this region the constraint on $|U_{e4}|$ versus $|U_{\mu4}|$ (Fig. 1(a)) has a slightly asymmetric shape for small values of $|U_{e4}|$ induced by the BABAR measurement of $BF(\tau \rightarrow \mu\nu\bar{\nu})/BF(\tau \rightarrow e\nu\bar{\nu})$ [40] which deviates by 1.8σ from the expected value of $PS(m_\mu, m_\tau)/PS(m_e, m_\tau) = 0.9726$ in the case of lepton universality. In contrast, the LEP I measurements of $BF(\tau \rightarrow e\nu\bar{\nu})$ and $BF(\tau \rightarrow \mu\nu\bar{\nu})$ are in very good agreement with the expectation from lepton universality: $BF(\tau \rightarrow \mu\nu\bar{\nu})/BF(\tau \rightarrow e\nu\bar{\nu}) = 0.9725 \pm 0.0043$. The constraint $|U_{e4}|$ versus $|U_{\tau4}|$ shows an asymmetry at small values as well (Fig. 1(e)) indicating a small but not significant deviation from lepton universality between the electron and τ sector. In contrast $|U_{\tau4}|$ versus $|U_{\mu4}|$ (Fig. 1(c)) shows an almost perfect symmetry at small values reflecting at the current level of precision very good agreement of the data with the lepton universality hypothesis between the μ and the τ sector.

3 The role of radiative decays $\mu \rightarrow e\gamma$, $\tau \rightarrow e\gamma$ and $\tau \rightarrow \mu\gamma$

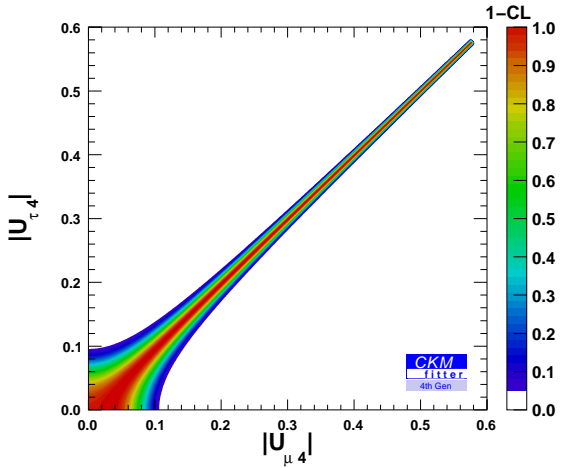
Observables which can provide constraints on products of PMNS matrix elements and hence introduce information which is “orthogonal” to the one obtained from leptonic decays are radiative μ - and τ -decay rates. We take the expression for the calculation of the radiative decay rates from Ref. [41]. In [41] the decay rate was calculated for a heavy third generation neutrino after the τ lepton had been discovered at SPEAR. To a very good approximation the result does not depend on the contributions from light neutrinos. By replacing the mixing parameters with corresponding PMNS matrix elements for the



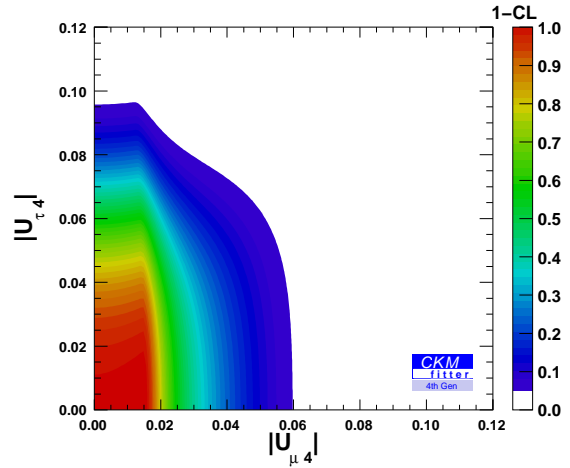
(a)



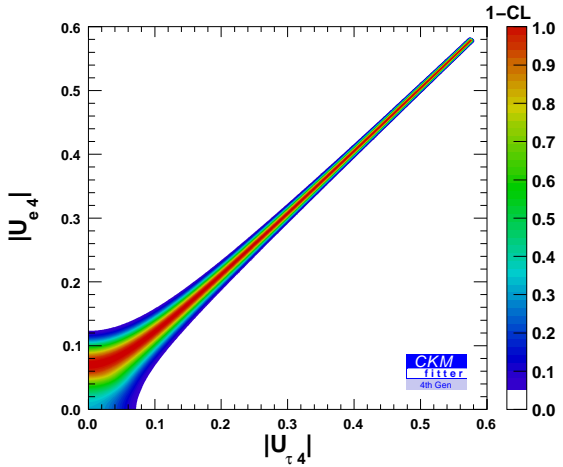
(b)



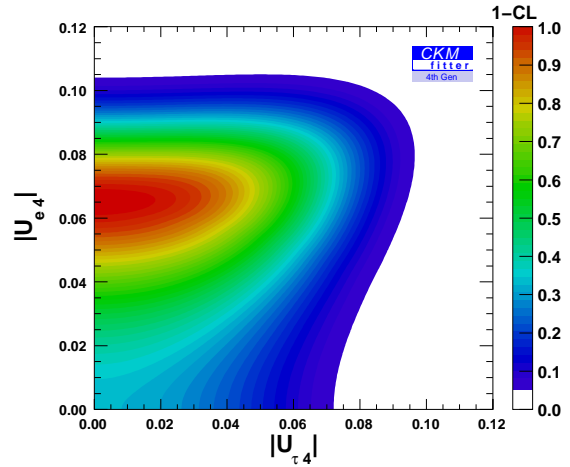
(c)



(d)



(e)



(f)

Figure 1: Two-dimensional constraints on $|U_{e4}|$, $|U_{\mu 4}|$ and $|U_{\tau 4}|$. Left column: results when leptonic μ and τ decays are combined. Right column: results when radiative μ and τ decay measurements are taken into account as well. Please note the different ranges on both axis. The color coding shows 1 - Confidence Level (1-CL)

4SM case one can write:

$$\Gamma(\mu^- \rightarrow e^- \gamma) = \frac{G_F^2 m_\mu^5}{192\pi^3} \cdot 3 \cdot \frac{\alpha(m_\mu) m_{\nu_4}^4}{32\pi m_W^4} \cdot |U_{e4}^* U_{\mu 4}|^2 \left[6(I^{(2)}\left(\frac{m_{\nu_4}^2}{m_W^2}\right) - I^{(3)}\left(\frac{m_{\nu_4}^2}{m_W^2}\right)) \right]^2, \quad (5)$$

$$\Gamma(\tau^- \rightarrow e^- \gamma) = \frac{G_F^2 m_\tau^5}{192\pi^3} \cdot 3 \cdot \frac{\alpha(m_\tau) m_{\nu_4}^4}{32\pi m_W^4} \cdot |U_{e4}^* U_{\tau 4}|^2 \left[6(I^{(2)}\left(\frac{m_{\nu_4}^2}{m_W^2}\right) - I^{(3)}\left(\frac{m_{\nu_4}^2}{m_W^2}\right)) \right]^2, \quad (6)$$

and

$$\Gamma(\tau^- \rightarrow \mu^- \gamma) = \frac{G_F^2 m_\tau^5}{192\pi^3} \cdot 3 \cdot \frac{\alpha(m_\tau) m_{\nu_4}^4}{32\pi m_W^4} \cdot |U_{\mu 4}^* U_{\tau 4}|^2 \left[6(I^{(2)}\left(\frac{m_{\nu_4}^2}{m_W^2}\right) - I^{(3)}\left(\frac{m_{\nu_4}^2}{m_W^2}\right)) \right]^2 \quad (7)$$

with m_{ν_4} being the mass of the heavy fourth generation neutrino and

$$I^{(n)}(x) = \int_0^1 \frac{dz \cdot z^n}{z + x \cdot (1 - z)}. \quad (8)$$

It is important to note that the decay rate for these radiative decays do not vanish in the limit $m_{\nu_4} \rightarrow \infty$ since the neutrino Yukawa-coupling increases with increasing m_{ν_4} which compensates the decrease of the loop integral. Therefore one can not escape the constraints from radiative lepton decays.

Since the mass limits from LEP2 on a fourth generation neutrino are only valid for unstable neutrinos [42] these limits depend implicitly on the mass of the heavy charged lepton and on the size of the PMNS matrix elements. To extract numerical results that are independent from these assumptions we choose for the heavy neutrino mass the robust constraint $m_{\nu_4} > m_Z/2$ resulting from the LEP1 measurements of the Z -resonance [17]. This is a conservative choice since for higher masses the constraints are getting stronger. For $m_{\nu_4} = 45$ GeV one finds $\left[6(I^{(2)}\left(\frac{m_{\nu_4}^2}{m_W^2}\right) - I^{(3)}\left(\frac{m_{\nu_4}^2}{m_W^2}\right)) \right]^2 \approx 0.513$.

We use the following experimental search limits: $BF(\mu \rightarrow e\gamma) < 1.2 \times 10^{-11}$ [43], $BF(\tau \rightarrow e\gamma) < 3.3 \times 10^{-8}$ [44], and $\tau \rightarrow \mu\gamma < 4.4 \times 10^{-8}$ [44] where all limits are quoted at 90 % CL. Figs. 1(b),1(d), and 1(f) show the constraints on $|U_{e4}|$ versus $|U_{\mu 4}|$, $|U_{e4}|$ versus $|U_{\tau 4}|$, and $|U_{\tau 4}|$ versus $|U_{\mu 4}|$ obtained by combining the leptonic μ and τ decays discussed in the previous section together with the search limits for the radiative decays $\mu \rightarrow e\gamma$, $\tau \rightarrow e\gamma$ and $\tau \rightarrow \mu\gamma$. The constraint from $BF(\mu \rightarrow e\gamma)$ imposes a constraint of the form $|U_{e4}| = \text{const.}/|U_{\mu 4}|$ corresponding to a hyperbolic shape. When combined with the constraint depicted in Fig. 1(a) this hyperbolic shape becomes clearly visible in Fig. 1(b). Due to the much weaker search limits for the radiative τ decays this particular shape is barely observable in Fig. 1(d) and not visible in Fig. 1(f). These plots demonstrate that the constraints from leptonic μ and τ decays when combined with the search limits from the radiative μ and τ decays are narrowed with respect to the leptonic μ and τ constraints only. The constraints lead to the following PMNS matrix:

$$U_{PMNS} = \begin{pmatrix} * & * & * & < 0.099 \\ * & * & * & < 0.048 \\ * & * & * & < 0.086 \\ < 0.122 & < 0.122 & < 0.122 & > 0.9925 \end{pmatrix} \quad (9)$$

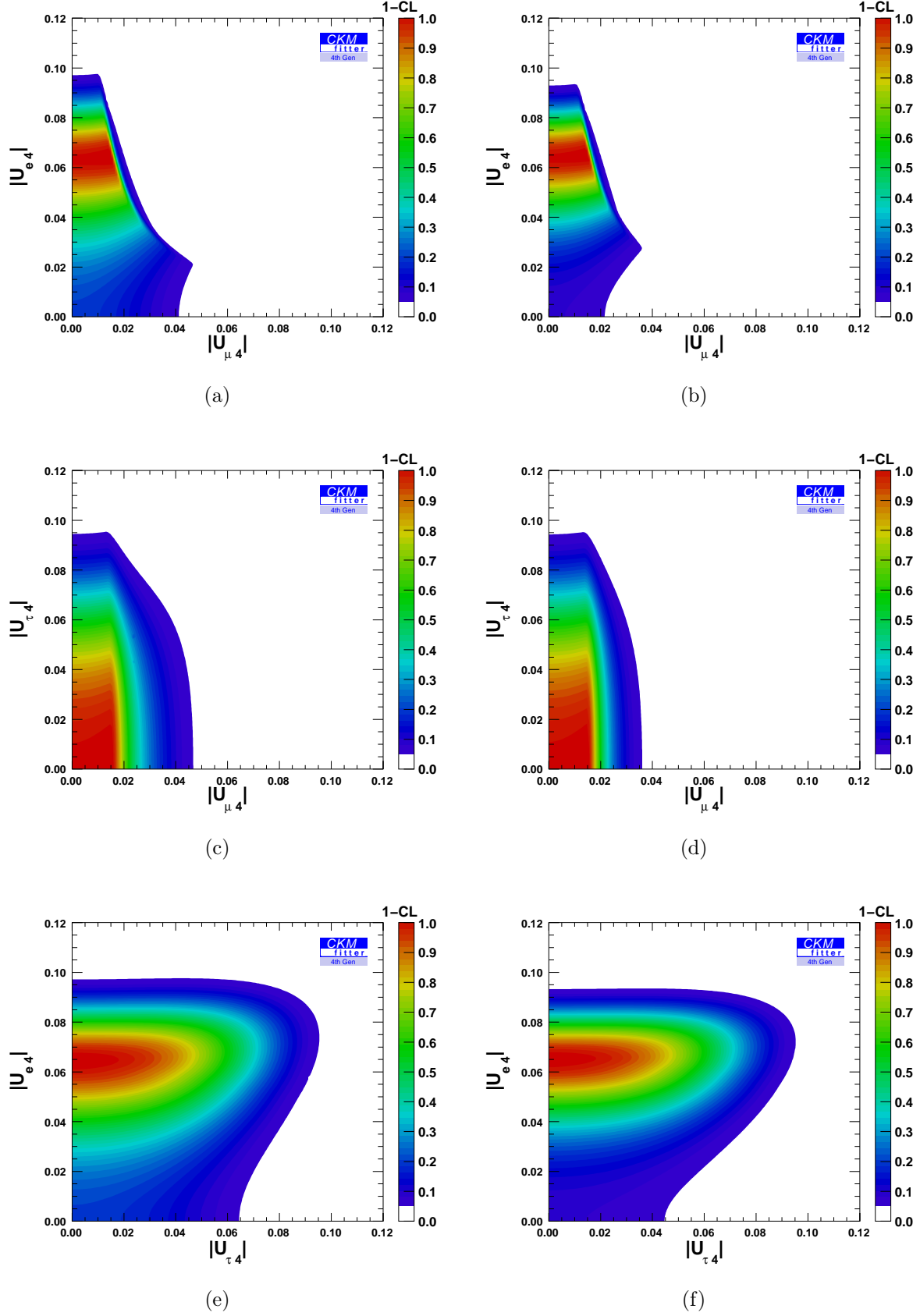


Figure 2: Two-dimensional constraints on $|U_{e4}|$, $|U_{\mu 4}|$ and $|U_{\tau 4}|$. Left column: results when leptonic and radiative μ , respectively, τ decays are combined with $K_{\ell 3}$ measurements. Right column: results when $\pi_{\ell 2}$ measurements are taken into account as well

where we quote 2σ limits and do not quantify any constraints on the 3×3 submatrix for the first three light neutrino generations which is determined by oscillation experiments. As a result of the improved constraints on the fourth generation PMNS matrix elements, the precision of G_F is considerably improved in the combined fit as well: $G_F^{3SM} \leq G_F^{4SM} \leq 1.0050 \cdot G_F^{3SM}$ (2σ level). That is, G_F could still be up to 0.50 % larger than the value obtained when assuming 3×3 unitarity for the first three generations in the neutrino sector.

Figs. 1(b), 1(d), and 1(f) also reveal that the precision on the fourth generation PMNS matrix elements is limited by how well lepton universality is tested by the measurement of the μ lifetime and the leptonic τ decays.

4 Leptonic and semileptonic decays of kaons and pions

In addition to leptonic μ and τ decays one can test lepton universality for example in leptonic and semileptonic kaon decays.

In the ratio of branching fractions $R_K = BF(K^+ \rightarrow e^+\nu_e)/BF(K^+ \rightarrow \mu^+\nu_\mu)$ the dependence on the CKM element $|V_{us}|^2$ as well as on G_F and other factors like the decay constant cancels and one can constrain $(1 - |U_{e4}|^2)/(1 - |U_{\mu4}|^2)$. The most precise value for R_K has been presented recently by the NA62 collaboration as a preliminary result with a relative uncertainty of 0.7 % [45]. This result has still a lower precision than the ratio of branching fractions between the corresponding leptonic τ decays (relative uncertainty of around 2.9 % when the LEP I and *BABAR* results are combined). When taking into account the new NA62 result the world average $R_K = (2.498 \pm 0.014)$ % [45] has a relative uncertainty of 0.56 % to be compared to the 3SM prediction of $R_K^{theo} = (2.477 \pm 0.001)$ % [46]. Since the NA62 result is still to be considered preliminary and the world average for R_K without this result has a quite large uncertainty of 0.97 % we do not use this input in our fit. The final relative uncertainty on R_K anticipated by NA62 is 0.4 % which should have a significant impact on lepton universality tests.

In contrast to leptonic kaon decays, semileptonic K_{e3} and $K_{\mu3}$ branching fractions provide already a test of lepton universality competitive with leptonic μ and τ decays [47]. The semileptonic kaon decay rates including radiative corrections are taken from Ref. [48] and modified for the fourth generation scenario as

$$\Gamma(K_{\ell3(\gamma)}) = \frac{G_F^2 m_K^5}{192\pi^3} C_K S_{ew} f_+(0)^2 |V_{us}|^2 I_K^\ell(\lambda_{+,0}) (1 + \delta_{SU(2)}^K + \delta_{em}^{K\ell})^2 \cdot (1 - |U_{\ell4}|^2), \quad (10)$$

where $\ell = e$ or μ , V_{us} is the CKM matrix element for the weak transition between a s- and u-quark, and C_K is a Clebsch-Gordan coefficient being equal to 1 ($1/\sqrt{2}$) for a neutral kaon in the initial state leading to a charged pion in the final state (for a charged kaon in the initial state leading to a neutral pion in the final state). $I_K^\ell(\lambda_{+,0})$ is a form-factor dependent phase integral where λ_+ and λ_0 are the form factor slope parameters for the charged, respectively, neutral kaon decay. S_{ew} is the short-distance electroweak correction calculated in Ref. [49]. $\delta_{SU(2)}^K$ is a long-distance isospin-breaking correction, and $\delta_{em}^{K\ell}$

takes into account the long-distance QED corrections [50]. The vector form-factor at zero-momentum transfer is denoted by $f_+(0)$.

By comparing $K_{\ell 3(\gamma)}$ decay rates for electron and muon final states for a charged, respectively, a neutral kaon in the initial state most of the factors cancel. The Flavianet Kaon Working Group provides results for the quantities $|V_{us}|f_+(0)$ separated between charged and neutral kaon initial states as well as for leptonic final states, and it is also possible to extract the uncorrelated and correlated uncertainties from the information provided in [47]. In the fourth generation scenario these quantities have to be reinterpreted as $|V_{us}|f_+(0)\sqrt{(1-|U_{\ell 4}|^2)}G_F^{4SM}/G_F^{3SM}$. With these additional inputs we obtain tightened constraints on $|U_{e4}|$, $|U_{\mu 4}|$, and $|U_{\tau 4}|$ as shown in Figs. 2(a), 2(c), and 2(e). One notes that the constraint on $|U_{\mu 4}|$ becomes significantly tighter than on $|U_{e4}|$. This is understood as follows: the extracted $|V_{us}|f_+(0)$ value for the K_{e3} decays is smaller than for $K_{\mu 3}$, although they are still in agreement at the 1σ level, which leads to a higher preferred value of $|U_{e4}|$ compared to $|U_{\mu 4}|$. The PMNS matrix has then the following 2σ limits:

$$U_{PMNS} = \begin{pmatrix} * & * & * & < 0.092 \\ * & * & * & < 0.037 \\ * & * & * & < 0.085 \\ < 0.117 & < 0.117 & < 0.117 & > 0.9932 \end{pmatrix}. \quad (11)$$

The allowed range for G_F is further tightened: $G_F^{3SM} \leq G_F^{4SM} \leq 1.0044 \cdot G_F^{3SM}$ (2σ level).

Analogous to leptonic Kaon decays the ratio of branching fractions for charged pions to leptonic final states ($\pi_{\ell 2}$ decays)

$$R_\pi = \frac{BF(\pi^+ \rightarrow e^+\nu_e)}{BF(\pi^+ \rightarrow \mu^+\nu_e)} \quad (12)$$

allows to constrain $(1-|U_{e4}|^2)/(1-|U_{\mu 4}|^2)$. Using the experimental measurements $BF(\pi^+ \rightarrow e^+\nu_e(\gamma)) = (1.230 \pm 0.004) \cdot 10^{-4}$ [38] (which is an average of the measurements quoted in Refs. [51–53]) and $BF(\pi^+ \rightarrow \mu^+\nu_\mu(\gamma)) = 0.9998770 \pm 0.0000004$ [38], and the theoretical prediction $R_\pi^{theo} = (1.2354 \pm 0.0002) \cdot 10^{-4}$ [54] we obtain the constraints on $|U_{e4}|$, $|U_{\mu 4}|$, and $|U_{\tau 4}|$ as shown in Figs. 2(b), 2(d), and 2(f).

For the PMNS matrix the 2σ limits read then:

$$U_{PMNS} = \begin{pmatrix} * & * & * & < 0.089 \\ * & * & * & > 0.021 \\ * & * & * & < 0.029 \\ < 0.115 & < 0.115 & < 0.115 & < 0.085 \\ & & & & < 0.9998 \\ & & & & > 0.9934 \end{pmatrix} \quad (13)$$

One should note the non-trivial lower limit for $|U_{e4}|$ and the non-trivial upper limit for $|U_{E4}|$ where we denote the heavy charged lepton as E . The p-value for the 3SM point $|U_{e4}| = 0$ is now 2.6 %. The allowed range for G_F is again tightened: $1.0002 \cdot G_F^{3SM} \leq G_F^{4SM} \leq 1.0040 \cdot G_F^{3SM}$ (2σ level). The lower limit on G_F^{4SM} being larger than the G_F^{3SM} value is directly related to the lower (upper) limit on $|U_{e4}|$ ($|U_{E4}|$). These correlations are illustrated in Fig. 3(a)-3(d) where we show the two-dimensional constraints on $|U_{e4}|$, $|U_{\mu 4}|$, $|U_{\tau 4}|$ and $|U_{E4}|$ as a function of G_F .

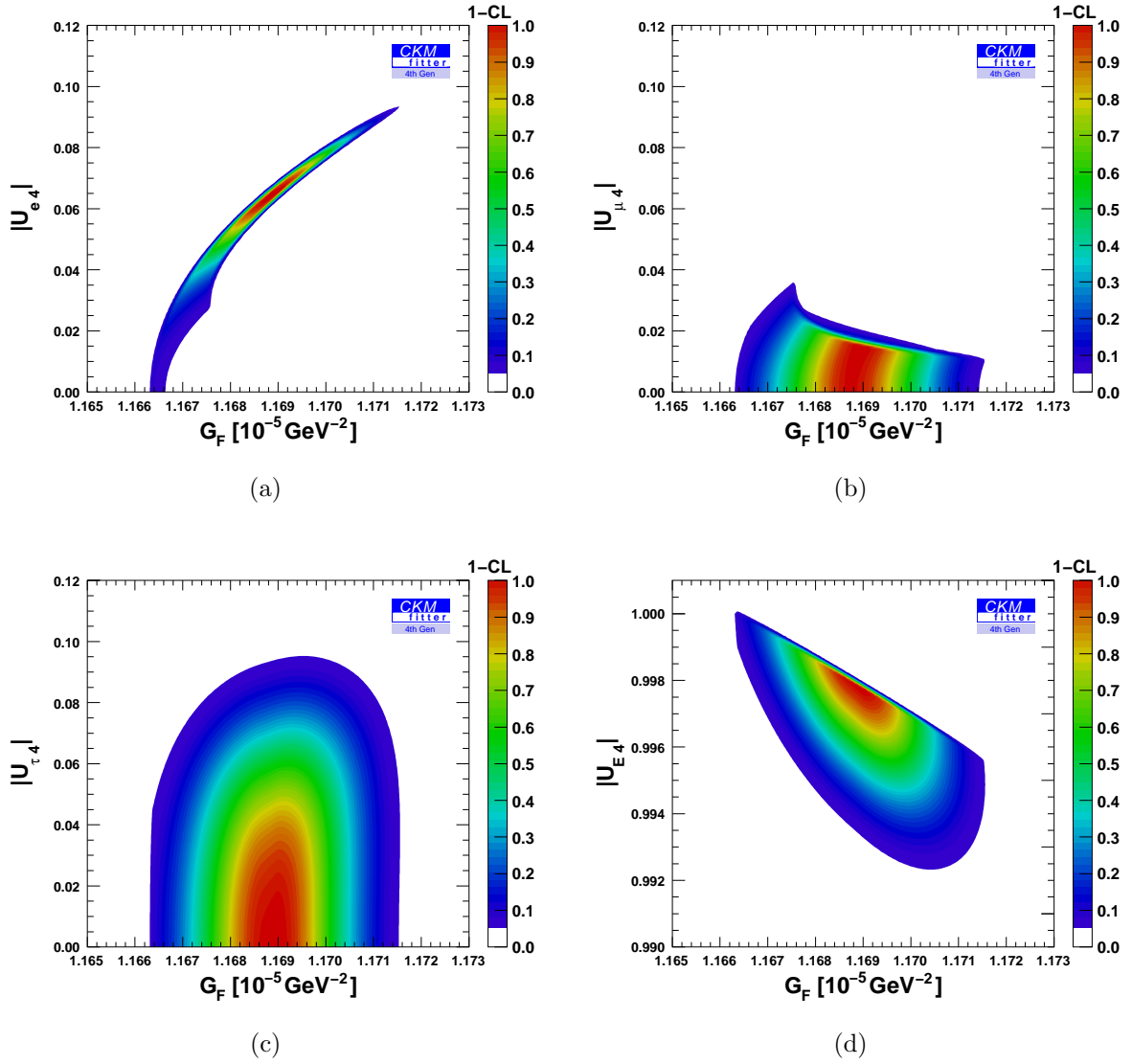


Figure 3: Two-dimensional constraints on $|U_{e4}|$, $|U_{\mu 4}|$, $|U_{\tau 4}|$ and $|U_{E4}|$ as a function of G_F .

5 Discussion

We discuss first the consequences related to the loss in precision in the extracted G_F value and the dependence of traditional CKM observables on PMNS matrix elements.

- CKM elements extracted from leptonic and semileptonic meson decays have to be separated according to the leptonic final state.
- Within a four generation scenario the extraction of $|V_{ud}|$ from superallowed β decays is significantly affected in its precision: one extracts $|V_{ud}|\sqrt{(1 - |U_{e4}|^2)}G_F^{4SM}/G_F^{3SM}$ instead of $|V_{ud}|$. In a recent analysis, Hardy and Towner find $|V_{ud}| = 0.97425 \pm 0.00022$ [55]. Interpreting this number as $|V_{ud}|\sqrt{(1 - |U_{e4}|^2)}G_F^{4SM}/G_F^{3SM}$ and performing a combined fit with all our inputs to constrain G_F^{4SM} and the four generation PMNS elements results in $|V_{ud}| = 0.97413_{-0.00023}^{+0.00033}$ at 1σ level, $|V_{ud}| = 0.97413_{-0.00046}^{+0.00053}$ at 2σ level and $|V_{ud}| = 0.97425_{-0.00100}^{+0.00071}$ at 3σ level. As a consequence, the precision in the extracted $|V_{ud}|$ value is significantly lower compared to the 3SM case. This reduction in precision for $|V_{ud}|$ will impact the constraints on fourth generation CKM elements imposed by 4×4 unitarity. Other constraints on CKM matrix elements are barely affected at the current level of precision.
- Ratios of CKM matrix elements determined from leptonic and semileptonic meson decays can be extracted without any change as long as measured rates are used with same final state leptons.
Also the ratio $R = |V_{tb}|^2/(|V_{td}|^2 + |V_{ts}|^2 + |V_{tb}|^2)$ extracted from $p\bar{p} \rightarrow t\bar{t}X$ events with zero, one or two b-tagged jets is unaffected.
- The CKM matrix elements $|V_{cd}|$ and $|V_{cs}|$ when extracted from deep inelastic scattering of ν_μ and $\bar{\nu}_\mu$ on nucleons can be determined without modifications. In this case, one measures the ratio of yields between dimuon and single muon events [56–58]. The muon common to both event classes originates from the charged current transition between the muon neutrino and the muon. As a consequence, the dependence on $1 - |U_{\mu 4}|^2$ cancels in the ratio and one extracts $|V_{cd}|^2 \times B_c = (4.63 \pm 0.34) \times 10^{-3}$, where $B_c = 0.0919 \pm 0.0094$ [59–61] is the measured branching fraction of semileptonic decays of charmed hadrons.
- Within the 4SM scenario the reduced precision in G_F requires a new study of the electroweak precision fit since the usual strategy in these fits is to neglect the uncertainty coming from G_F . In such a fit the dependence of the electroweak precision observables on PMNS and CKM matrix elements in the self-energy terms needs to be taken into account. Observables like the W -boson mass or the partial rate $\Gamma(Z \rightarrow l^+l^-)$ depend at leading order on G_F . Only at higher order do the fourth generation particles enter with a corresponding dependence on CKM and PMNS matrix elements. In turn, a consistent fourth generation electroweak precision fit might strengthen the constraint on G_F .

Next, we discuss the bounds found for the fourth generation PMNS matrix elements in more detail.

- In the past there have been studies on fourth generation PMNS matrix elements using leptonic τ decays at a time when there was a discrepancy between the corresponding branching fraction values, the τ lifetime and the τ mass [62]. Only after an improved determination of the τ mass leading to a significant shift in the measured value made the discrepancy vanish.

In the analysis of Ref. [62] it was assumed that there is only mixing between the third and fourth generation. Hence, the determination of G_F from the muon lifetime was considered to be unaffected by the presence of a fourth generation.

As a consequence, our analysis is much more general: it is able to extract limits on all PMNS matrix elements related to the fourth generation and to determine simultaneously the Fermi constant G_F .

- In Ref. [11] the radiative decay $\mu \rightarrow e\gamma$ was used, assuming $G_F = G_F^{3SM}$, to set a limit on $|U_{e4}U_{\mu 4}^*|$. However, no limit on the individual fourth generation PMNS elements could be set in Ref. [11] since the leptonic μ and τ decays have not been considered. To obtain the limit of $|U_{e4}U_{\mu 4}^*| < 4 \cdot 10^{-4}$ the authors of Ref. [11] used the search limit for the heavy neutrino mass, $m_{\nu_4} = 90.3$ GeV [42]. Within our analysis the limit on this product is significantly looser, $|U_{e4}U_{\mu 4}^*| < 1.16 \cdot 10^{-3}$ (2σ level), since we conservatively take $m_{\nu_4} > m_Z/2$. It has to be stressed though that our individual limits on $|U_{e4}|$, $|U_{\mu 4}|$, and $|U_{\tau 4}|$ would not change very much if $m_{\nu_4} > 90.3$ GeV were used in our analysis.
- Recently, *BABAR* has presented measurements of $BF(\tau \rightarrow h\nu_\tau)$ ($h = \pi, K$) [40]. When compared to $BF(h \rightarrow \mu\nu_\mu)$ *BABAR* finds a 3σ deviation from lepton universality in the $\mu - \tau$ sector. Interpreted in a four generation scenario the *BABAR* result [40] reads $\sqrt{1 - |U_{\tau 4}|^2}/\sqrt{1 - |U_{\mu 4}|^2} = 0.985 \pm 0.005$. In contrast, we find $\sqrt{1 - |U_{\tau 4}|^2}/\sqrt{1 - |U_{\mu 4}|^2} = 1.0001_{-0.0017}^{+0.0001}$ (1σ level) in our analysis which does not use the $\tau \rightarrow h\nu_\tau$ data. As a consequence, we do not observe such a deviation from lepton universality.
- Leptonic W decays $W \rightarrow \ell\nu_\ell$ ($\ell = e, \mu, \tau$) can be used as well to test lepton universality. However, the experimental precision of the measured branching fractions $BF(W \rightarrow e\nu_e) = 0.1075 \pm 0.0013$ [38], $BF(W \rightarrow \mu\nu_\mu) = 0.1057 \pm 0.0015$ [38], $BF(W \rightarrow \tau\nu_\tau) = 0.1125 \pm 0.0020$ [38], can not compete with the other lepton universality tests studied in our analysis. It is important to note though that for leptonic W decays the preferred but not significant hierarchy is $|U_{\tau 4}| < |U_{e4}| < |U_{\mu 4}|$. As a consequence, including the leptonic W branching fractions does not change significantly the constraints due to the lower precision but leads to an increase of the χ^2 value of the combined fit from 4.1 (without W decays) to 10.7 (including W decays).
- The constraints on the fourth generation PMNS matrix elements obtained from our study allow to draw several interesting conclusions:
 - In Ref. [63] a unification of spins and charges has been considered to explain family replication. A specific symmetry breaking mechanism has been discussed in [63] leading to the following prediction for the fourth generation PMNS

matrix:

$$U_{PMNS} = \begin{pmatrix} 0.697 & 0.486 & 0.177 & 0.497 \\ 0.486 & 0.697 & 0.497 & 0.177 \\ 0.177 & 0.497 & 0.817 & 0.234 \\ 0.497 & 0.177 & 0.234 & 0.817 \end{pmatrix}. \quad (14)$$

By only considering the PMNS elements of the fourth row and column we conclude that our analysis clearly rules out the PMNS matrix predicted by this specific symmetry breaking mechanism within the unification approach of spin and charges.

- In typical see-saw models [64–66] one expects that the mixing angle between the fourth and the third generation θ_{34} fulfills the relation $\sin^2 \theta_{34} \approx m_\tau/m_E$ where m_τ is the τ -lepton mass and m_E the mass of the fourth-generation charged lepton. Using the LEP search limit, $m_E > 100$ GeV at 90 % CL [42], one finds $\sin \theta_{34} < 0.13$. Since $\sin \theta_{34} \approx |U_{\tau 4}|$ (in the Botella-Chau parametrisation [31]) our findings are in agreement with such a scenario if m_E is at least of order 240 GeV.
- While each individual system (leptonic τ decays, $K_{\ell 3}$ decays, and leptonic π decays) shows no significant deviation from lepton universality between electrons and muons it is interesting to note that the deviation seen in each case goes into the same direction and points to a non-zero value for $|U_{e4}|$. It should be stressed though that neither the individual nor the combined constraints are sufficient to claim evidence for a significant deviation of $|U_{e4}|$ from zero. If confirmed by more stringent tests of lepton universality this would point to New Physics. In case of a fourth generation it would imply an interesting hierarchy: $|U_{e4}| > |U_{\mu 4}|$ with a surprisingly large value for $|U_{e4}|$. To this end we note that the preliminary world average $R_K = (2.498 \pm 0.014)$ % [45] for leptonic Kaon decays, if confirmed, would reduce the observed deviation of $|U_{e4}|$ from zero.
- In Ref. [15] it was studied whether a heavy fourth generation neutrino could explain the discrepancy between the measured and predicted value of the anomalous magnetic moment of the muon. It was concluded that this could be only the case if the PMNS matrix element $|U_{\mu 4}|$ were of order 0.7. The authors consider such a large value unrealistic as it has to fulfill the search limit $BF(\mu \rightarrow e\gamma) < 1.2 \times 10^{-11}$ [43] which in turn requires the element $|U_{e4}|$ to be very tiny at the same time.

Our analysis which takes into account radiative and leptonic μ and τ decays allows us now to draw a firm conclusion. Using the calculated contribution from a heavy fourth generation neutrino as given in Ref. [15] one is not able to explain the discrepancy between the measured and predicted value for the anomalous magnetic moment of the muon since we find $|U_{\mu 4}| < 0.029$ (2σ level) which is much smaller than the required size of $O(0.7)$.

6 Summary

Recent analyses of elements of the neutrino mass and quark mass mixing matrices as well as of electroweak precision fits within the framework of a fourth generation implicitly assume 3×3 unitarity for the first three generations in the neutrino sector or at least assume no mixing between the fourth and first two neutrino generations. When relaxing this ad-hoc assumption we find that the value of the Fermi constant G_F from the μ -lifetime measurement can only be extracted with low precision and only when using also leptonic τ branching fractions. However, the leptonic μ and τ decays test lepton universality and thereby lead to strong correlations between the fourth generation PMNS matrix elements $|U_{e4}|$, $|U_{\mu4}|$, and $|U_{\tau4}|$. The precision on G_F can only be improved significantly if one adds the search limits from radiative μ and τ decays which in turn allows to set interesting constraints on the individual elements $|U_{e4}|$, $|U_{\mu4}|$, and $|U_{\tau4}|$. Still, the precision on G_F is at least two orders of magnitude worse compared to the 3SM value. $K_{\ell 3}$ decays and leptonic decays of charged pions provide similar stringent tests of lepton universality and thereby improve further the constraints on $|U_{e4}|$, $|U_{\mu4}|$, $|U_{\tau4}|$, and on G_F .

In our generalized analysis we find that the CKM sector is not very much affected with one exception: the precision on $|V_{ud}|$ extracted from super-allowed β decays is reduced. This reduction in precision will impact the constraints on fourth generation CKM elements imposed by 4×4 unitarity.

Interestingly, the limits obtained on $|U_{e4}|$, $|U_{\mu4}|$, and $|U_{\tau4}|$ are able to exclude already certain models that predict patterns for a fourth generation neutrino mixing matrix. In this context, it is interesting to note that we observe a p-value of 2.6 % for $|U_{e4}| = 0$. If this deviation could be confirmed with more precise tests of lepton universality it would indicate New Physics and possibly point to the existence of a fourth generation although such a large mixing between the fourth and first generation looks surprising.

We do not observe any violation of lepton universality between the τ and the μ sector in contrast to an evidence of such as found by *BABAR* in a recent measurement of $BF(\tau \rightarrow h\nu_\tau)$ ($h = \pi, K$) when compared to $BF(h \rightarrow \mu\nu_\mu)$.

With $|U_{\mu4}| < 0.029$ found in our analysis we are able to exclude, using the study of Ref. [15], the possibility that the discrepancy between the measured and predicted value for the anomalous magnetic moment of the muon can be explained by a contribution of a heavy fourth generation neutrino.

Future studies of fourth generation parameter constraints (PMNS matrix, CKM matrix and electroweak precision fit) need to relax the ad-hoc assumption of 3×3 unitarity for the first three generations. The authors are currently preparing studies within the CKMfitter package that combine the PMNS sector, the CKM sector, and the electroweak precision fit in a consistent way.

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